Rappels sur les Dérivées

I Calculer les différentielles des fonctions suivantes:

1) $y(x) = \frac{1}{x}$; $y(x) = \frac{1}{x^2}$; $y(x) = \frac{1}{x^3}$; $y(x) = \frac{1}{x-a}$; $y(x) = \frac{1}{a-x}$; $y(x) = \frac{k}{x^{5/2}}$;

$V(r) = \frac{q}{4\pi\varepsilon_0 r}$

2) $y(x) = \frac{1}{x^2 + a^2}$; $y(x) = \frac{1}{\left(x^2 + a^2\right)^{3/2}}$; $y(x) = \frac{1}{\left(a^2 - x^2\right)^{5/2}}$; $E(z) = \frac{az}{\left(z^2 + a^2\right)^{3/2}}$

3) $y(x) = (2 - x)^4$; $y(x) = \left(2 + x^2\right)^3$

4) $y(x) = 3x^3\left(2 - 4x^4\right)^5$; $y(x) = \frac{x}{1 + x^2}$; $y(x) = \frac{\left(1 - x^2\right)^3}{\left(1 + 4x^3\right)^2}$; $y(x) = x\left(1 + \frac{2}{1 - x}\right)$;

$y(x) = \sqrt{1 - \frac{ax}{x^2 + a^2}}$

5) $y(x) = \sin(2x)$; $y(x) = \sin\left(\frac{x}{2}\right)$; $y(x) = \cos^2\left(\frac{x}{2}\right)$; $y(x) = \sin^2\left(\frac{x}{2}\right)$; $y(x) = \cos^2\left(\frac{x^2}{2}\right)$;

$y(x) = \sin^2\left(\frac{x^2}{2}\right)$; $y(x) = \tan\left(x^3\right)$; $y(x) = \cos\left(x^3\right)$; $y(x) = \sin^2(x)\cos(x)$;

$y(x) = \sqrt{(1 - \cos(3x))^3}$; $y(x) = \sin^2\left(4x^2 - 1\right)$; $n(A) = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

6) $y(x) = \ln(1 - x)^2$; $y(x) = \ln\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)$; $y(x) = e^{-x^2}\left(3x^2 + 6x - 1\right)$; $y(x) = \frac{1}{\sin x}$;

$V(x) = \frac{\lambda}{4\pi\varepsilon}\ln(1 - x)$

7) $y(x) = \arccos(x)$; $y(x) = \arcsin(x)$; $y(x) = \arctan(x)$; $y(x) = \ln(x)$

8) $y(x) = \left(x^2 + 1\right)^2$; $y(t) = \cos^3(\omega t)$; $y(x) = \ln\left(2x^2\right)$; $\gamma(t) = gt^2 + at + 1$;

$f(t) = t^2\left(3t^2 + 2t + 1\right)$; $y(x) = \frac{bx}{b^2 + x^2}$; $V(x,y) = \frac{k}{\sqrt{x^2 + y^2}}$; $\gamma(h,h') = \frac{h}{h - h'}$;

$R(\theta_1,\theta_2) = \rho\frac{\theta_2 - \theta_1}{\theta_1}$

II Calculer les différentielles logarithmiques des fonctions suivantes:

$y(x) = \left(x^2 + 1\right)^2$; $y(x) = \frac{x}{x + 1}$; $f(t) = t^2\left(3t^2 + 2t + 1\right)$; $V(x,y) = \frac{k}{\left(x^2 + y^2\right)^{1/2}}$

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