Mécanique Vibratoire

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1: Théorème du moment d'inertie : $\sum \mathcal{M}_{ext} = I_{Ox} \ddot{\theta}$

$$\sum \mathcal{M}_{ext} = -g \underbrace{\left(m x + m_L \frac{L}{2} - Md - m_d \frac{d}{2} \right)}_{Z} \sin\theta - c(\theta - \theta_0)$$

$$\rightarrow \quad I_{Ox} \ddot{\theta} + g Z \sin\theta + c(\theta - \theta_0) = 0$$

$$\ddot{\theta} + \frac{gZ}{I_{Ox}} \sin\theta + \frac{c}{I_{Ox}} (\theta - \theta_0) = 0$$

petites oscillat : $\ddot{\theta} = \ddot{\varepsilon}$; $\theta = \varepsilon$; $\sin\theta = \varepsilon$

$$\rightarrow \quad \boxed{\ddot{\varepsilon} + \left(\frac{gZ + c}{I_{Ox}} \right) \varepsilon + \cancel{\frac{c\,\theta_0}{I_{Ox}}} = 0}$$

équilibre : $\ddot{\varepsilon} = 0$; $\dot{\varepsilon} = 0$
avec $\varepsilon = 0$
$\Rightarrow \theta_0 = 0$

2: $$\omega_0^2 = \frac{gZ + c}{I_{Ox}} = \left(\frac{2\pi}{T_0} \right)^2$$

$$\boxed{T_0 = 2\pi \sqrt{\frac{I_{Ox}}{gZ + c}}}$$

$d = 1$; $x = \frac{3}{2}$; $L = 2$
$M = 1$; $m = \frac{1}{2}$; $m_L = 1$; $m_d = \frac{1}{2}$
$\Big\}$ $Z = 1$

$I_{Ox} = 11$; $c = 1$; $g = 10$

$$\rightarrow \boxed{T_0 = 2\pi \ \text{s}}$$

en utilisant le Th E cinétique :

$$\boxed{\frac{dE_c}{dt} = \mathcal{P}(\vec{F}_{ext})} \qquad E_c = \frac{1}{2} I_{Ox} \dot{\theta}^2$$

$$= I_{Ox} \dot{\theta} \ddot{\theta}$$

Forces : $\vec{R}(O)$; $\vec{P}(B)$; $\vec{P}(C)$; $\vec{P}_{OD}\left(\frac{L}{2}\right)$; $\vec{P}_{OB}\left(\frac{d}{2}\right)$; $\vec{\mathcal{M}}_0$

$$\mathcal{P}(\vec{R}(O)) = \vec{R} \cdot \vec{v}(O) = \vec{0} \qquad (\vec{v}(O) = \vec{0})$$

$$\mathcal{P}(\vec{P}(B)) = \vec{P}(B) \cdot \vec{v}(B) \qquad \vec{v}(B) = \frac{d\vec{OB}}{dt} = \dot{\theta} d \left(\cos\theta\, \vec{e}_y + \sin\theta\, \vec{e}_z\right)$$

$$= -Mgd\dot{\theta}\sin\theta \qquad \vec{P}(B) = -Mg\,\vec{e}_z$$

$$\mathcal{P}(\vec{P}(C)) = \vec{P}(C) \cdot \vec{v}(C) \qquad v(C) = \frac{d\vec{OC}}{dt} = -\dot{\theta} x \left(\cos\theta\, \vec{e}_y + \sin\theta\, \vec{e}_z\right)$$

$$= mgx\dot{\theta}\sin\theta \qquad \vec{P}(C) = -mg\,\vec{e}_z$$

$$\mathcal{P}\left(\vec{P}_{OD}\left(\tfrac{L}{2}\right)\right) = m_L g \frac{L}{2} \dot{\theta} \sin\theta$$ analogue à $\mathcal{P}(\vec{P}(C))$

$$\mathcal{P}\left(\vec{P}_{OB}\left(\tfrac{d}{2}\right)\right) = -m_d g \frac{d}{2} \dot{\theta} \sin\theta$$ " $\mathcal{P}(\vec{P}(B))$

$$\mathcal{P}(\vec{\mathcal{M}}_0) = \mathcal{M}_0 \dot{\theta} = c(\theta - \theta_0)\dot{\theta}$$

$$I_{Ox} \dot{\theta}\ddot{\theta} + g\dot{\theta}\sin\theta \left(Md - mx + m_d \frac{d}{2} - m_L \frac{L}{2}\right) - c\theta\dot{\theta} = 0$$

→ id précédemment établi avec Th du moment.

En utilisant le Th de l'E mécanique

$$\boxed{\frac{dE}{dt} = \mathcal{P}_{nc}(\vec{F})} = 0 \quad \text{(pas de forces non-conservatives)}$$

$$E_c = \frac{1}{2} I_{Ox} \dot{\theta}^2 \quad \rightarrow \quad \frac{dE_c}{dt} = I_{Ox} \dot{\theta}\ddot{\theta}$$

$E_{pOB} = -m_d\, g \frac{d}{2} \cos\theta$

$E_{pOD} = m_L\, g \frac{L}{2} \cos\theta$

$E_p(c) = m\, g\, x \cos\theta$

$E_p(B) = -M g d \cos\theta$

origine des E_p en $z = 0$

$$\frac{dE_p}{dt} = g\dot{\theta} \sin\theta \left(m_d \frac{d}{2} - m_L \frac{L}{2} - m x + M d \right)$$

$$E_p(\text{ressort spiral}) = -\frac{1}{2} c (\theta - \theta_0)^2 \quad \rightarrow \quad \frac{dE_p(\text{ressort})}{dt} = -c(\theta - \theta_0)^2$$

$$I_{Ox} \dot{\theta}\ddot{\theta} + g\dot{\theta} \sin\theta \left(m_d \frac{d}{2} - m_L \frac{L}{2} - m x + M d \right) - c\theta = 0$$

→ id. préc.