3) 31) $$\frac{dE}{dt} = \mathcal{P}_{non.\,cons}$$

$$E_c = E_{c\,OA} + E_{c\,AB} = \frac{1}{2} I_{OA}\,\dot\theta^2 + \frac{1}{2} m v_A^2$$
$$= \frac{1}{6} m L^2 \dot\theta^2 + \frac{1}{2} m L^2 \dot\theta^2 = \frac{2}{3} m L^2 \dot\theta^2$$

$$E_p = E_{p\,ressort\,linéaire} + E_{p\,OA} + E_{p\,AB} + E_{p\,ressort\,spiral}$$
$$= \frac{1}{2} k (l - l_0)^2 + mg\frac{L}{2}\sin\theta + mgL\sin\theta + \frac{1}{2} c\,\theta^2 \qquad l = d - L\sin\theta$$

$$\frac{dE_c}{dt} = \frac{1}{3} m L^2 \dot\theta\ddot\theta + m L^2 \dot\theta\ddot\theta = \frac{4}{3} m L^2 \dot\theta\ddot\theta$$
$$\frac{dE_p}{dt} = k(d - L\sin\theta - l_0)(-L\dot\theta\cos\theta) + \frac{3}{2} mgL\dot\theta\cos\theta + c\,\theta\dot\theta$$
$$\mathcal{P}_{n.c.} = \mathcal{P}_{ressort\,spiral} = -\chi\dot\theta^2$$

$$\frac{4}{3} m L^2 \cancel{\dot\theta}\ddot\theta + \frac{3}{2} mgL\cancel{\dot\theta}\cos\theta + c\,\theta\cancel{\dot\theta} + k(d - L\sin\theta - l_0)(-L\cancel{\dot\theta}\cos\theta) + \chi\dot\theta^{\cancel{2}} = 0$$

$$\frac{4}{3} m L^2 \ddot\theta + \chi\dot\theta + c\,\theta + \left[\frac{3}{2} mgL - kL(d - L\sin\theta - l_0)\right]\cos\theta = 0$$

faibles amplitudes de mouvements : $\theta = \varepsilon \quad \cos\theta = 1 \quad \sin\theta = \varepsilon$
$\dot\theta = \dot\varepsilon \quad \ddot\theta = \ddot\varepsilon$

$$\frac{4}{3} m L^2 \ddot\varepsilon + \chi\dot\varepsilon + (kL^2 + c)\,\varepsilon + \frac{3}{2} mgL - kL(d - l_0) = 0$$

32) équilibre : $\dot{\theta} = 0 \,;\, \ddot{\theta} = 0 \implies \dot{\varepsilon} = 0 ,\, \ddot{\varepsilon} = 0$

$\theta = 0 \rightarrow \varepsilon = 0$

$$\implies \frac{3}{2} mgL - kL(d - l_0) = 0$$

$$d = \frac{3mgL}{kL} + l_0$$

$$\boxed{d = \frac{3}{2}\frac{mg}{k} + l_0}$$

$$\implies \boxed{\frac{4}{3} m L^2 \ddot{\varepsilon} + \chi \dot{\varepsilon} + (kL^2 + c)\varepsilon = 0}$$

33) L'équa° différentielle se met sous la forme :

$$\ddot{\varepsilon} + 2\xi\omega_0 \dot{\varepsilon} + \omega_0^2 \varepsilon = 0$$

$$\ddot{\varepsilon} + \frac{3}{4}\frac{\chi}{mL^2}\dot{\varepsilon} + \frac{3}{4}\frac{kL^2 + c}{mL^2}\varepsilon = 0$$

avec $\boxed{\omega_0 = \frac{1}{2}\left(3\frac{kL^2 + c}{mL^2}\right)^{1/2}}$

et $2\xi\omega_0 = \frac{3}{4}\frac{\chi}{mL^2} \implies \boxed{\xi = \frac{3}{8\omega_0}\frac{\chi}{mL^2}}$