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1) m en translation : $E_C = \frac{1}{2} m \dot{x}_3^2 = \frac{1}{2} m v^2(C)$

$$x_3 = l \sin\theta \simeq l\theta$$

$$v^2(C) = (l\dot\theta)^2 \quad \rightarrow \quad E_C = \frac{1}{2} m l^2 \dot\theta^2$$

$$E_p = E_{p1} + E_{p2} + E_p(\text{masse})$$

$$E_{p1} = \frac{1}{2} k_1 x_1^2 = \frac{1}{2} k_1 \left(\frac{l}{3}\sin\theta\right)^2 \sim \frac{1}{2} k_1 \left(\frac{l}{3}\theta\right)^2$$

$$E_{p2} = \frac{1}{2} k_2 x_2^2 = \frac{1}{2} k_2 \left(\frac{2l}{3}\sin\theta\right)^2 \sim \frac{1}{2} k_2 \left(\frac{2l}{3}\theta\right)^2$$

$$E_p(\text{masse}) = -mgl\cos\theta$$

$$E_p = \frac{1}{2} k_1 \frac{l^2}{9}\theta^2 + \frac{1}{2} k_2 \frac{4l^2}{9}\theta^2 - mgl\cos\theta$$

$$\mathcal{P}(F(C)) = -\alpha \vec{v}(C) \cdot \vec{v}(C) = -\alpha v^2(C) = -\alpha(l\dot\theta)^2$$

$$\frac{dE}{dt} = \mathcal{P}_{nc}(\vec{F}_{ext}) = \mathcal{P}(\vec{F}(C)) = -\alpha(l\dot\theta)^2$$

$$\cancel{m}\, l g \cancel{\dot\theta} \sin\theta + m \cancel{l^2} \cancel{\dot\theta}\ddot\theta + \frac{2}{2} k_1 \frac{\cancel{l^2}}{9} \cancel{\dot\theta}\theta + k_2 \frac{4\cancel{l^2}}{9}\cancel{\dot\theta}\theta = -\alpha l^2 \dot\theta^{\cancel{2}}$$

$$\frac{mg}{l}\theta + m\ddot\theta + \frac{k_1}{9}\theta + \frac{4k_2}{9}\theta = -\alpha\dot\theta$$

$$\frac{g}{l}\theta + \ddot\theta + \frac{1}{9m}(k_1 + 4k_2)\theta + \frac{\alpha}{m}\dot\theta = 0$$

2) de la forme $\ddot\theta + 2\xi\omega_0\dot\theta + \omega_0^2\theta = 0$

$$\omega_0 = \sqrt{\frac{1}{9m}(k_1 + 4k_2) + \frac{g}{l}} \simeq \sqrt{\frac{1}{9 \cdot 10}(5000 + 4000)} \simeq 10 \text{ rad s}^{-1}$$

$$2\xi\omega_0 = \frac{\alpha}{m} \quad \rightarrow \quad \xi = \frac{\alpha}{2\omega_0 m} = 0{,}01$$

3) Il faudrait rajouter l'énergie cinétique de rotation de la barre $E_c = \frac{1}{2} I \dot\theta^2$, et son énergie potentielle de pesanteur $E_p = Mg\frac{L}{2}\sin\theta$

Par les 3 théorèmes principaux

$$\vec{OC} = l \sin\theta \, \vec{e_x} - l\cos\theta \, \vec{e_y}$$

$$\vec{v}_A = \begin{vmatrix} \frac{l}{3}\dot\theta\cos\theta \\ \frac{l}{3}\dot\theta\sin\theta \end{vmatrix} \quad \vec{v}_B = \begin{vmatrix} \frac{2l}{3}\cos\theta \\ \frac{2l}{3}\sin\theta \end{vmatrix}$$

$$\vec{v}(c) = \frac{d\vec{OC}}{dt} = l\dot\theta\cos\theta\,\vec{e_x} + l\dot\theta\sin\theta\,\vec{e_y}$$

$$v^2(c) = (l\dot\theta)^2$$

$$E_p(\text{masse}) = -mgl\cos\theta$$

$$E_c(\text{masse}) = \frac{1}{2} m v^2(c) = \frac{1}{2} m (l\dot\theta)^2$$

$$I_{Oz}(\text{masse}) = m l^2$$

$$\frac{dE_c}{dt} = m l^2 \dot\theta \ddot\theta \qquad \frac{dE_p(\text{masse})}{dt} = mgl\dot\theta\sin\theta$$

$$E_{p1} = \frac{1}{2}k_1\left(\frac{l}{3}\theta\right)^2$$

$$E_{p2} = \frac{1}{2}k_2\left(\frac{2l}{3}\theta\right)^2$$

$$\frac{dE_{p1}}{dt} = k_1\left(\frac{l}{3}\right)^2\theta\dot\theta \qquad \frac{dE_{p2}}{dt} = k_2\left(\frac{2l}{3}\right)^2\theta\dot\theta$$

$$\mathcal{P}(\vec{F_c}) = -\alpha(l\dot\theta)^2$$

$$\mathcal{P}(\text{masse}) = -mgl\dot\theta\theta$$

$$\vec{\mathcal{M}}_0(\vec{P}(c)) = \begin{vmatrix} l\sin\theta \\ -l\cos\theta \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -mg \end{vmatrix} = -mgl\sin\theta\,\vec{e_z} \simeq -mgl\theta\,\vec{e_z}$$

$$\vec{\mathcal{M}}_0(\vec{T}(A)) = \vec{OA}\wedge\vec{T}(A) = \left(\frac{l}{3}\sin\theta\,\vec{e_x} - \frac{l}{3}\cos\theta\,\vec{e_y}\right)\wedge\left(-k_1x_1\vec{e_x}\right)$$

$$= -\frac{l}{3}k_1x_1\cos\theta\,\vec{e_z} \simeq -\frac{l}{3}k_1x_1\vec{e_z} = -\left(\frac{l}{3}\right)^2 k_1\theta\,\vec{e_z}$$

$$\vec{\mathcal{M}}_0(\vec{T}(B)) = -\frac{2l}{3}k_2x_2\cos\theta\,\vec{e_z} \simeq -\frac{2l}{3}k_2x_2\,\vec{e_z} = -\left(\frac{2l}{3}\right)^2 k_2\theta\,\vec{e_z}$$

$$\vec{\mathcal{M}}_0(\vec{F}(c)) = \vec{OC}\wedge\vec{F_c} = \begin{vmatrix} l\sin\theta \\ -l\cos\theta \end{vmatrix} \wedge \begin{vmatrix} -\alpha l\dot\theta\cos\theta \\ -\alpha l\dot\theta\sin\theta \end{vmatrix} = -\alpha l^2\dot\theta$$

$$\mathcal{P}(\vec{T}(A)) = \vec{T}(A)\cdot\vec{v}(A) = \left(-k_1x_1\vec{e_x}\right)\cdot\left(\frac{l}{3}\dot\theta\cos\theta\,\vec{e_x} + \frac{l}{3}\dot\theta\cos\theta\,\vec{e_y}\right)$$

$$= -k_1x_1\frac{l}{3}\dot\theta\cos\theta \simeq -k_1x_1\frac{l}{3}\dot\theta = -k_1\left(\frac{l}{3}\right)^2\theta\dot\theta$$

$$\mathcal{P}(\vec{T}(B)) = \vec{T}(B)\cdot\vec{v}(B) = -k_2x_2\frac{2l}{3}\cos\theta \simeq -k_2\left(\frac{2l}{3}\right)^2\theta\dot\theta$$

$$\mathcal{P}(\vec{F}(c)) = \vec{F}(c)\cdot\vec{v}(c) = -\alpha v_c^2 = -\alpha(l\dot\theta)^2$$

Théorème de l'énergie mécanique totale : $\boxed{\frac{dE}{dt} = \mathcal{P}_{nc}(\vec{F}_{ext})}$

$$\ddot\theta + \left[\frac{1}{9m}(k_1 + 4k_2) + \frac{g}{l}\right]\theta + \frac{\alpha}{m}\dot\theta = 0$$

Théorème de l'énergie cinétique : $\boxed{\frac{dE_c}{dt} = \mathcal{P}(\vec{F}_{ext})}$

$$m\cancel{l^2}\cancel{\dot{\theta}}\ddot{\theta} = -k_1\left(\frac{\cancel{l}}{3}\right)^2\theta\cancel{\dot{\theta}} - k_2\left(\frac{2\cancel{l}}{3}\right)^2\theta\cancel{\dot{\theta}} - \alpha(\cancel{l}\dot{\theta})^{\cancel{2}} - mg\frac{\cancel{l}}{l}\cancel{\dot{\theta}}\theta$$

$$m\ddot{\theta} + k_1\frac{1}{9}\theta + k_2\frac{4}{9}\theta + \alpha\dot{\theta} + \frac{mg}{l}\theta = 0$$

$$\ddot{\theta} + \left[\frac{1}{9m}(k_1 + 4k_2) + \frac{g}{l}\right]\theta + \frac{\alpha}{m}\dot{\theta} = 0$$

Théorème du moment cinétique $\boxed{\mathcal{M}_\Delta = I_{Oz}\ddot{\theta}}$ $\Delta = Oz$

$$ml^2\ddot{\theta} = -\left(\frac{l}{3}\right)^2k_1\theta - \left(\frac{2l}{3}\right)^2k_2\theta - \alpha l^2\dot{\theta} - mgl\theta$$

$$ml^2\ddot{\theta} + \left(\left(\frac{l}{3}\right)^2k_1 + \left(\frac{2l}{3}\right)^2k_2 + mgl\right)\theta + \alpha l^2\dot{\theta} = 0$$

$$\ddot{\theta} + \frac{\alpha}{m}\dot{\theta} + \left[\frac{k_1 + 4k_2}{9m} + \frac{g}{l}\right]\theta = 0$$