TD Mécanique n°1

-1- 1) $\vec{OM} \cdot \vec{e_x} = r \cos(\vec{OM}, \vec{e_x}) = r \cos(-\theta) \quad (= r\cos(\vec{e_x}, \vec{OM})) = r\cos\theta$

$\vec{OM} \cdot \vec{e_y} = r \cos(\vec{OM}, \vec{e_y}) = r \cos[(\vec{OM}, \vec{e_x}) + (\vec{e_x}, \vec{e_y})]$

$= r \cos(-\theta + \frac{\pi}{2}) = r \sin\theta$

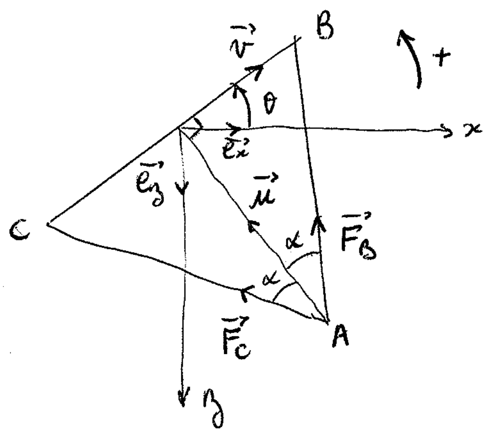
$\longrightarrow \quad \vec{OM} = r\cos\theta \, \vec{e_x} + r \sin\theta \, \vec{e_y}$

2) $r\cos\theta$ est la projecᵒ orthogonale de $\vec{OM}$ sur $\vec{Ox}$

$r\sin\theta$ ——— ——— ——— Oy

Il faut faire attention aux signes si angle négatifs.

-2-



$(\vec{F_B}, \vec{e_x}) = (\vec{F_B}, \vec{u}) + (\vec{u}, \vec{v}) + (\vec{v}, \vec{e_x})$

$= \alpha - \frac{\pi}{2} - \theta$

$(\vec{F_B}, \vec{e_z}) = (\vec{F_B}, \vec{e_x}) + (\vec{e_x}, \vec{e_z})$

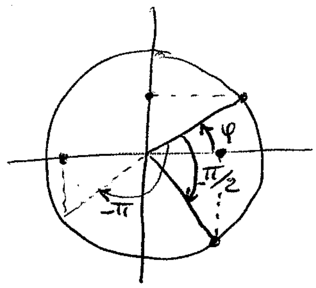
$= \alpha - \frac{\pi}{2} - \theta - \frac{\pi}{2}$

$= \alpha - \theta - \pi$

$\vec{F_B} = F_B \cos(\vec{F_B}, \vec{e_x}) \vec{e_x} + F_B \cos(\vec{F_B}, \vec{e_z}) \vec{e_z}$

$= F_B \cos(\alpha - \frac{\pi}{2} - \theta) \vec{e_x} + F_B \cos(\alpha - \theta - \pi) \vec{e_z}$

$= F_B \sin(\alpha - \theta) \vec{e_x} - F_B \cos(\alpha - \theta) \vec{e_z}$



$(\vec{F_C}, \vec{e_x}) = (\vec{F_C}, \vec{F_B}) + (\vec{F_B}, \vec{e_x})$

$= -2\alpha + \alpha - \frac{\pi}{2} - \theta = -\alpha - \frac{\pi}{2} - \theta$

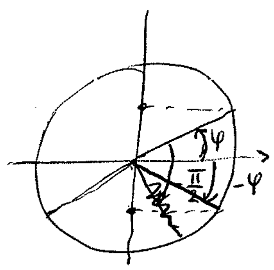
$(\vec{F_C}, \vec{e_z}) = (\vec{F_C}, \vec{F_B}) + (\vec{F_B}, \vec{e_z})$

$= -2\alpha + \alpha - \theta - \pi = -\alpha - \theta - \pi$

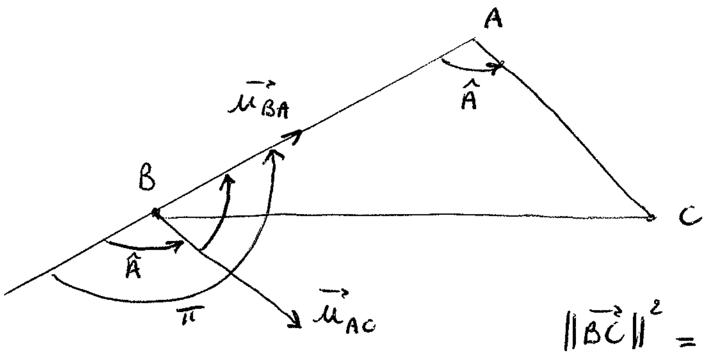
$\vec{F_C} = F_C \cos(-\alpha - \theta - \frac{\pi}{2}) \vec{e_x} + F_C \cos(-\alpha - \theta - \pi) \vec{e_z}$

$= F_C \sin(-\alpha - \theta) \vec{e_x} - F_C \cos(-\alpha - \theta) \vec{e_z}$

$= -F_C \sin(\alpha + \theta) \vec{e_x} - F_C \cos(\alpha + \theta) \vec{e_z}$



- 3 -



$$\|\vec{BC}\|^2 = (\vec{BC})^2 = (\vec{BA} + \vec{AC})^2$$
$$= \|\vec{BA}\|^2 + \|\vec{AC}\|^2 + 2\,\vec{BA}\cdot\vec{AC}$$

$$\vec{BA}\cdot\vec{AC} = \|\vec{BA}\|\,\|\vec{AC}\| \cos(\vec{BA}\cdot\vec{AC})$$
$$= cb\cos(\pi - \hat{A}) = -cb\cos\hat{A}$$

$$\rightarrow \quad \boxed{a^2 = c^2 + b^2 - 2cb\cos\hat{A}}$$

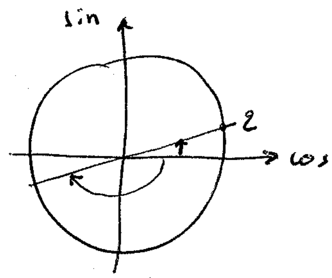
- 4 - **Polairs :** $x = r\cos\theta$ $\quad y = r\sin\theta$

On projette les vecteurs sur la base polaire (plan (xoy)) :

$$\vec{OP'} = 2\vec{e_x} + 4\vec{e_y}$$
$$\vec{OQ'} = -\vec{e_x} - 5\vec{e_y}$$

pour P : $\left.\begin{array}{l} r\cos\theta = 2 \\ r\sin\theta = 4 \end{array}\right\} \rightarrow \tan\theta = 2 \rightarrow \left\{\begin{array}{l} r^2 = 16 + 4 = 20 \quad \boxed{r = 2\sqrt{5}} \\ \theta = \arctan 2 + k\pi \end{array}\right.$

$\rightarrow$ 2 angle possibles : soit ds le quartier $[0, 90]$, soit dans le quartier $[-90°, -180°]$



ici $\left|\begin{array}{l} \cos\theta > 0 \\ \sin\theta > 0 \end{array}\right\}$ 1er quartier

$$\Rightarrow \boxed{\theta = \arctan 2 = 63{,}5°}$$

pour Q : $\left.\begin{array}{l} r\cos\theta = -1 \\ r\sin\theta = -5 \end{array}\right\} \begin{array}{l} \tan\theta = 5 \\ r^2 = 26 \end{array} \left.\right\} \begin{array}{l} \boxed{r = \sqrt{26}} \\ \left.\begin{array}{l} \cos\theta < 0 \\ \sin\theta < 0 \end{array}\right\} \boxed{\theta = \arctan 5 \mp \pi} \end{array}$

$(-101{,}3°$ ou $258{,}7°)$

Cylindrique : les $\hat{m}$ + $z = z$

$$\vec{OP}\cdot\vec{OQ} = \begin{vmatrix} 2 \\ 4 \\ 3 \end{vmatrix} \cdot \begin{vmatrix} -1 \\ -5 \\ 4 \end{vmatrix} = -2 - 20 + 12 = -10$$

$$\vec{OP}\cdot\vec{OQ} = \|\vec{OP}\| \cdot \|\vec{OQ}\| \cos(\vec{OP}, \vec{OQ})$$

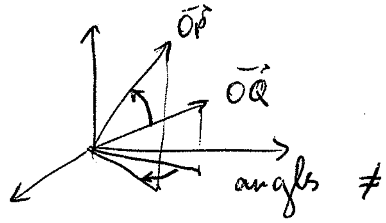
$$\rightarrow \quad \cos(\vec{OP}, \vec{OQ}) = \frac{\vec{OP}\cdot\vec{OQ}}{\|\vec{OP}\|\|\vec{OQ}\|}$$

$$\left.\begin{aligned} \|\vec{OP}\| &= \sqrt{4+16+9} = \sqrt{29} \\ \|\vec{OQ}\| &= \sqrt{1+25+16} = \sqrt{42} \end{aligned}\right\} \cos(\vec{OP}, \vec{OQ}) = \frac{-10}{\sqrt{29}\sqrt{42}}$$

$$=$$

les 2 angles ayant ce cosinus sont $\arccos\left(\frac{-10}{\sqrt{29}\sqrt{42}}\right) = \pm$

Rq : cet angle est $\neq$ de l'angle entre les 2 vecteurs proj. orthogonalement sur (Ox, Oy) :



-5- 1) $\vec{A}\wedge\vec{B}$ amène $\vec{A}$ sur $\vec{B}$ par rotation. En appliquant le sens direct $\vec{A} \rightarrow \vec{B} \rightarrow \underline{\underline{-\vec{e_z}}}$

donc $\vec{A}\wedge\vec{B} = -AB\sin(\theta_1+\theta_2)\,\vec{e_z}$

2) Angles orientés par $\vec{e_z}$: $\vec{A}\wedge\vec{B} = AB\sin(\vec{A},\vec{B})\,\vec{e_z}$

$(\vec{A},\vec{B}) = -\theta_1 + \theta_2$ (rotation de $\vec{A} \rightarrow \vec{B}$)

$\rightarrow \vec{A}\wedge\vec{B} = AB\sin(-\theta_1+\theta_2)\,\vec{e_z} = -AB\sin(\theta_1-\theta_2)\,\vec{e_z}$

3) Angles orientés par $-\vec{e_z}$: $\vec{A}\wedge\vec{B} = AB\sin(\vec{A},\vec{B})(-\vec{e_z})$

$= AB\sin(-\theta_1+\theta_2)(-\vec{e_z})$

$= -AB\sin(-\theta_1+\theta_2)\,\vec{e_z}$

-6- Soient $\overrightarrow{OP} = A_1 \vec{e_x} + A_2 \vec{e_y} + A_3 \vec{e_z}$ et $\overrightarrow{OQ} = B_1 \vec{e_x} + B_2 \vec{e_y} + B_3 \vec{e_z}$

$$\overrightarrow{OP} \wedge \overrightarrow{OQ} = (A_1\vec{e_x} + A_2\vec{e_y} + A_3\vec{e_z}) \wedge (B_1\vec{e_x} + B_2\vec{e_y} + B_3\vec{e_z})$$

$$= A_1B_2\,\vec{e_z} - A_1B_3\,\vec{e_y} - A_2B_1\,\vec{e_z} + A_2B_3\,\vec{e_x} + A_3B_1\,\vec{e_y} - A_3B_2\,\vec{e_x}$$

$$= (A_2B_3 - A_3B_2)\vec{e_x} + (A_3B_1 - A_1B_3)\vec{e_y} + (A_1B_2 - A_2B_1)\vec{e_z}$$

Comparons :

$$\det\begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = \vec{e_x}(A_2B_3 - B_2A_3) + A_1(B_2\vec{e_z} - \vec{e_y}B_3) + B_1(\vec{e_y}A_3 - A_2\vec{e_z})$$

$$\text{———} \quad + (A_3B_1 - A_1B_3)\vec{e_y} + (A_1B_2 - A_2B_1)\vec{e_z}$$

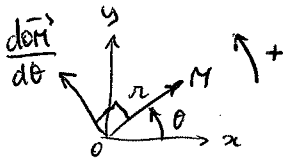
$\vec{e_y}$ $\vec{e_z}$

$$\left(\overrightarrow{OP} \wedge \overrightarrow{OQ}\right)_{-4-} = \begin{vmatrix} \vec{e_x} & \vec{e_y} & \vec{e_z} \\ 2 & 4 & 3 \\ -1 & -5 & 4 \end{vmatrix} = \vec{e_x}(16+15) - 2(4\vec{e_y} + 5\vec{e_z}) - 1(3\vec{e_y} - 4\vec{e_z})$$

$$= 31\,\vec{e_x} - 11\,\vec{e_y} - 6\,\vec{e_z}$$

Vérif : $\overrightarrow{OP} \wedge \overrightarrow{OQ} = \begin{vmatrix} 2 \\ 4 \\ 3 \end{vmatrix} \wedge \begin{vmatrix} -1 \\ -5 \\ 4 \end{vmatrix} = \begin{vmatrix} 31 \\ -11 \\ -6 \end{vmatrix}$

-7-



- $\overrightarrow{OM} = r\cos\theta\,\vec{e_x} + r\sin\theta\,\vec{e_y}$
- $\dfrac{d\overrightarrow{OM}}{d\theta} = -r\sin\theta\,\vec{e_x} + r\cos\theta\,\vec{e_y}$

$$= +r\cos\left(\theta + \frac{\pi}{2}\right)\vec{e_x} + r\sin\left(\theta + \frac{\pi}{2}\right)\vec{e_y}$$

$\dfrac{d\overrightarrow{OM}}{d\theta}$ décalé de $+\dfrac{\pi}{2}$, $\left\|\dfrac{d\overrightarrow{OM}}{d\theta}\right\| = \|\overrightarrow{OM}\|$

- $\dfrac{d\overrightarrow{OM}}{dt} = \dfrac{d\overrightarrow{OM}}{d\theta}\,\dfrac{d\theta}{dt}$

$$= \left(-r\sin\theta\,\vec{e_x} + r\cos\theta\,\vec{e_y}\right)\frac{d\theta}{dt}$$

$$= -r\dot{\theta}\sin\theta\,\vec{e_x} + r\dot{\theta}\cos\theta\,\vec{e_y}$$

=

$$\vec{\Omega} \wedge \overrightarrow{OM} = \dot{\theta}\,\vec{e_z} \wedge (r\cos\theta\,\vec{e_x} + r\sin\theta\,\vec{e_y})$$

$$= r\dot{\theta}\cos\theta\,\vec{e_y} - r\dot{\theta}\sin\theta\,\vec{e_x}$$

-8-

$$\vec{v} = \dot{r}\,\vec{e_r} + r\dot{\theta}\,\vec{e_\theta}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{r}\,\vec{e_r} + \dot{r}\,\frac{d\vec{e_r}}{dt} + \dot{r}\dot{\theta}\,\vec{e_\theta} + r\ddot{\theta}\,\vec{e_\theta} + r\dot{\theta}\,\frac{d\vec{e_\theta}}{dt}$$

$$\frac{d\vec{e_r}}{dt} = \frac{d\vec{e_r}}{d\theta}\,\frac{d\theta}{dt} = \vec{e_\theta}\cdot\dot{\theta}$$

$$\frac{d\vec{e_\theta}}{dt} = \frac{d\vec{e_\theta}}{d\theta}\,\frac{d\theta}{dt} = -\vec{e_r}\cdot\dot{\theta}$$

$\rightarrow \quad \vec{a} = \ddot{r}\,\vec{e_r} + 2\dot{r}\dot{\theta}\,\vec{e_\theta} + r\ddot{\theta}\,\vec{e_\theta} - r(\dot{\theta})^2\,\vec{e_r}$

m^{vt} circulaire : $r =$ cte $= R$

$r =$ cte $\rightarrow \dfrac{dr}{dt} = 0 \; (\dot{r} = 0)$ et $\ddot{r} = 0$

$\rightarrow \quad \vec{a} = R\ddot{\theta}\,\vec{e_\theta} - R\dot{\theta}^2\vec{e_r}$

m^{vt} circulaire uniforme : $\dot{\theta} =$ cte et $r =$ cte $\Rightarrow \dot{r} = 0 \quad \ddot{r} = 0 \quad \ddot{\theta} = 0$

$\rightarrow \quad \vec{a} = -R\dot{\theta}^2\,\vec{e_r}$