Méca Avril 2023

1) Trajectoire

11) $x(t) = v_0 \cos\alpha \, t \longrightarrow t = \dfrac{x}{v_0 \cos\alpha}$

$y(t) = -\dfrac{gt^2}{2} + v_0 \sin\alpha \, t$

$\rightarrow \quad y = -\dfrac{g}{2} \dfrac{x^2}{v_0^2 \cos^2\alpha} + \dfrac{v_0 \sin\alpha}{v_0 \cos\alpha} x$

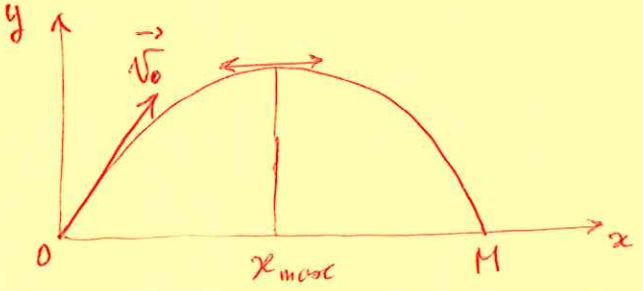
$$\boxed{y = -\frac{g}{2v_0^2 \cos^2\alpha} x^2 + \tan\alpha \, x}$$

12) $\vec{v}(P) = \begin{vmatrix} \dfrac{dx}{dt} \\ \dfrac{dy}{dt} \end{vmatrix} = \begin{vmatrix} v_0 \cos\alpha \\ -gt + v_0 \sin\alpha \end{vmatrix}$

$\vec{a}(P) = \begin{vmatrix} \dfrac{dv_x}{dt} \\ \dfrac{dv_y}{dt} \end{vmatrix} = \begin{vmatrix} 0 \\ -g \end{vmatrix}$

à $t=0 \quad \vec{v}(P) = \begin{vmatrix} v_0 \cos\alpha \\ v_0 \sin\alpha \end{vmatrix} = \begin{vmatrix} v_{0x} \\ v_{0y} \end{vmatrix}$

13)



$$\frac{dy}{dx} = -\frac{g}{v_0^2 \cos^2\alpha} x + \tan\alpha$$

$\dfrac{dy}{dx} = 0 \quad$ ssi $\quad x = \dfrac{\tan\alpha \cdot v_0^2 \cdot \cos^2\alpha}{g} = \dfrac{v_0^2}{2g} \sin 2\alpha = x_{max}$

14)

$$y(x) = 0 = x\left(-\frac{g}{2v_0^2\cos^2\alpha}x + \tan\alpha\right)$$

$\Rightarrow$ 2 solutions : $\begin{cases} x = 0 & \text{point de lancer de P : O} \\ x = \dfrac{2v_0^2\cos^2\alpha\tan\alpha}{g} = \dfrac{v_0^2}{g}\sin 2\alpha & \text{point d'arrivée de P au sol : M} \end{cases}$

OM est max pour $\sin 2\alpha$ max, soit $\sin 2\alpha = 1$

$\Rightarrow 2\alpha = \frac{\pi}{2}$ $\rightarrow$ $\boxed{\alpha = \frac{\pi}{4}}$