Février 2015

1) 11) $\vec{OM}(t) = r(t)\,\vec{e_r}(t) + z(t)\,\vec{e_z}$ /1

12) $$\vec{v}(M) = \frac{d\vec{OM}}{dt} = \frac{dr}{dt}\vec{e_r} + r\frac{d\vec{e_r}}{dt} + \frac{dz}{dt}\vec{e_z}$$

$$= \frac{dr}{dt}\vec{e_r} + r\frac{d\theta}{dt}\vec{e_\theta} + \frac{dz}{dt}\vec{e_z}$$

$$= \dot{r}\,\vec{e_r} + r\dot{\theta}\,\vec{e_\theta} + \dot{z}\,\vec{e_z}$$ /2

13) $$\vec{a}(M) = \frac{d\vec{v}(M)}{dt}$$

$$= \frac{d^2 r}{dt^2}\vec{e_r} + \left(\frac{dr}{dt}\dot{\theta} + r\frac{d\dot{\theta}}{dt}\right)\vec{e_\theta} + \frac{d\dot{z}}{dt}\vec{e_z} + \dot{r}\frac{d\vec{e_r}}{dt} + r\dot{\theta}\frac{d\vec{e_\theta}}{dt}$$

$$= \ddot{r}\,\vec{e_r} + (\dot{r}\dot{\theta} + r\ddot{\theta})\vec{e_\theta} + \ddot{z}\,\vec{e_z} + \dot{r}\dot{\theta}\,\vec{e_\theta} + r\dot{\theta}(-\dot{\theta}\,\vec{e_r})$$

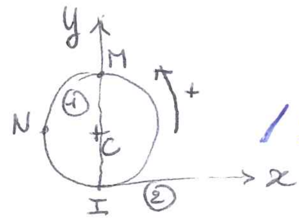
$$= (\ddot{r} - r\dot{\theta}^2)\vec{e_r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\vec{e_\theta} + \ddot{z}\,\vec{e_z}$$ /3

2) Solide S_1 : roue

Solide S_2 : sol $\longrightarrow$ $\vec{v}(I_2) = \vec{0}$

$$\vec{g}(S_1/S_2) = \vec{v}(I_1) - \vec{v}(I_2) = \vec{0}$$

$$\Rightarrow \vec{v}(I_1) = \vec{0}$$ /1

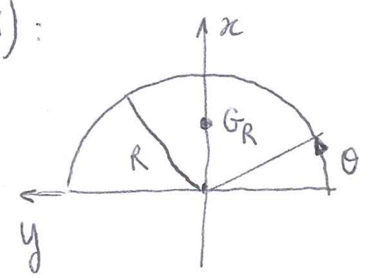


$$\vec{v}(C) = \vec{\Omega} \wedge \vec{IC} = \omega \vec{e_z} \wedge R\,\vec{e_y}$$
$$= -\omega R\,\vec{e_x}$$ /2

$$\vec{v}(M) = \vec{\Omega} \wedge \vec{IM} = -2\omega R\,\vec{e_x}$$ /1

$$\vec{v}(N) = \vec{v}(C) + \vec{\Omega} \wedge \vec{CN} = -\omega R\,\vec{e_x} + \omega\,\vec{e_z} \wedge (-R)\,\vec{e_x}$$
$$= -\omega R\,\vec{e_x} - \omega R\,\vec{e_y}$$ /2

3) 31) :



$m\vec{OG} = \int \vec{OM}\, dm$ /1

symétrie /, $Ox \rightarrow G_R \in Ox$ /1

$\Rightarrow y_{G_R} = 0$

$\Rightarrow m\, x_{G_R} = \int x\, dm$ $\qquad dm = \lambda R\, d\theta$

$x = R \sin\theta$ /1

$$m\, x_{G_R} = \lambda R^2 \int_0^{\pi} \sin\theta\, d\theta = 2\lambda R^2$$

$m = \pi R \lambda \quad \Rightarrow \quad \boxed{x_{G_R} = \frac{2R}{\pi}}$ /1

Ou par Guldin : $S = 4\pi R^2 = 2\pi\, L\, x_{G_R} = 2\pi \cdot \pi R\, x_{G_R}$

$\rightarrow x_{G_R} = \frac{2R}{\pi}$

de même : $\boxed{x_{G_r} = \frac{2r}{\pi}}$ /1

32)

$$(m+M)\,\overrightarrow{OG} = M\,\overrightarrow{OG_R} + m\,\overrightarrow{OG_r}$$ /1

Ox reste axe de symetrie : $G \in Ox$

$$\overrightarrow{OG} = x_G\,\vec{e_x} \quad ; \quad \overrightarrow{OG_R} = \frac{2R}{\pi}\,\vec{e_x} \quad ; \quad \overrightarrow{OG_r} = \frac{2r}{\pi}\,\vec{e_x}$$

/1

$$(m+M)\,x_G\,\cancel{\vec{e_x}} = M\cdot\frac{2R}{\pi}\,\cancel{\vec{e_x}} + m\cdot\frac{2r}{\pi}\,\cancel{\vec{e_x}}$$

$$\pi\cancel{\lambda}(r+R)\,x_G = \cancel{\pi}\cancel{\lambda}R\cdot\frac{2R}{\cancel{\pi}} + \cancel{\pi}\cancel{\lambda}r\cdot\frac{2r}{\cancel{\pi}}$$

$$x_G = \frac{2\,(R^2+r^2)}{\pi\,(R+r)}$$ /1

Guldin ? :

$$4\pi R^2 + 4\pi r^2 = 2\pi\,x_G\cdot(\pi R + \pi r)$$

$$x_G = \frac{4\pi\,(R^2+r^2)}{2\pi^2\,(R+r)} = \frac{2\,(R^2+r^2)}{\pi\,(R+r)}$$