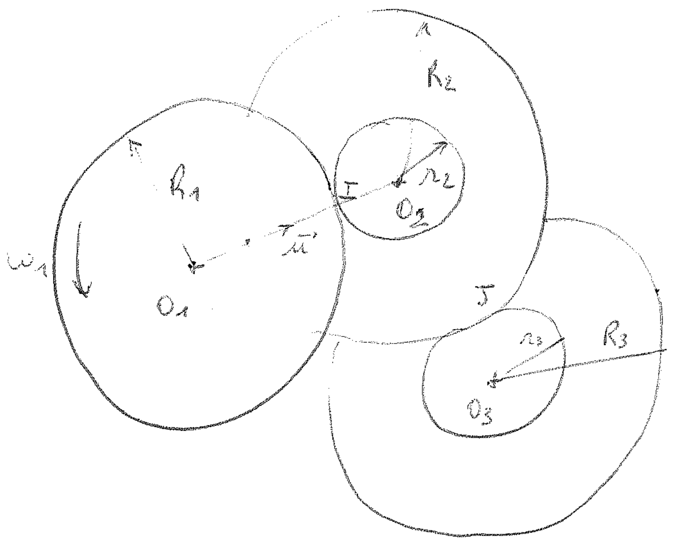
Exam 2009 : II



axes de rotations fixes
contact permanent sans glissement en I et J

relat $\omega_1, \omega_2, \omega_3$?

?

$$\vec{v}(I_1) = \vec{v}(I_2) \qquad\qquad \vec{v}(J_2) = \vec{v}(J_3)$$

$$\vec{v}(I_1) = \vec{\Omega}_1 \wedge \overrightarrow{O_1 I_1} + \cancel{\vec{v}(O_1)}$$
$$= \omega_1 \vec{e}_z \wedge R_1 \vec{u}$$
$$= \omega_1 R_1 \vec{v}$$
$$\vec{v}(I_2) = \vec{\Omega}_2 \wedge \overrightarrow{O_2 I_2} + \cancel{\vec{v}(O_2)}$$
$$= \omega_2 \vec{e}_z \wedge -r_2 \vec{u}$$
$$= -r_2 \omega_2 \vec{v}$$

$$\boxed{\omega_1 r_1 = - \omega_2 R_2}$$

$$\omega_1 R_1 = -\omega_2 r_2$$

$$\omega_3 = -\frac{\omega_2 r_2}{R_3} = \omega_1 \frac{r_1 r_2}{R_2 R_3}$$

de même : $\omega_3 r_3 = - \omega_2 R_2$

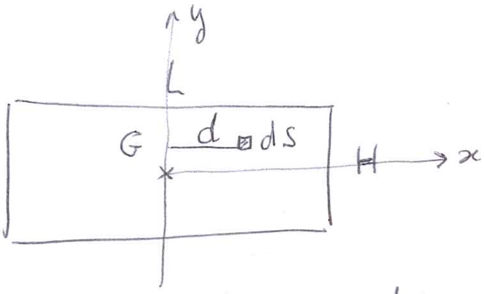
$$\boxed{\omega_2 r_2 = -\omega_3 R_3}$$

$$\Rightarrow \quad \omega_3 = -\frac{\omega_2 R_2}{r_3} = \frac{\omega_1 R_1 R_2}{r_2 r_3}$$

$$\omega_1 = \frac{r_2 r_3}{R_1 R_2} \omega_3$$

si $r_1 = r_i = r_2$ $= r$ et $R_1 = R_i = R_2$ $= R$: $\omega_f = \frac{r^2}{R^2} \omega_i$

2) 21)



$$I_{Gy} = \int d^2\, dm \qquad dm = \sigma\, dS = \sigma\, dx\, dy$$

$$d = x$$

$$= \sigma \int x^2 dx \int dy$$

$$= \sigma \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2 dx \int_{-\frac{H}{2}}^{\frac{H}{2}} dy = \sigma \left[\frac{x^3}{3}\right]_{-\frac{L}{2}}^{\frac{L}{2}} \left[y\right]_{-\frac{H}{2}}^{\frac{H}{2}}$$

$$\boxed{I_{Gy} = \sigma \frac{L^3}{12} H}$$

22) plaque pleine de dimensions $l \times h$: $I'_{Gy} = \sigma \frac{l^3}{12} h$

$$\Rightarrow \quad I_{Gy} = \frac{\sigma}{12}\left(L^3 H - l^3 h\right)$$

$$m = \sigma S = \sigma (LH - lh)$$

$$\Rightarrow \quad \boxed{I_{Gy} = \frac{m}{12} \frac{(L^3 H - l^3 h)}{(LH - lh)}}$$

23)

$$I_{Oy} = I_{Gy} + \left(\frac{L}{2}\right)^2 m$$

$$\boxed{I_{Oy} = \frac{m}{12}\left[\frac{L^3 H - l^3 h}{LH - lh}\right] + \frac{m}{4} L^2}$$

24)

$$I_{Oy} = \frac{20}{12}\left[\frac{0{,}5 - \frac{0{,}4}{8}}{0{,}5 - \frac{0{,}4}{2}}\right] + \frac{20}{4} = \frac{5}{3}\left(\frac{0{,}45}{0{,}3}\right) + 5 = \frac{5}{3} \cdot \frac{3}{2} + 5 = \frac{15}{2}\ \text{kg.m}^2$$

3)

3.1) $$\frac{dE}{dt} = \mathcal{P}_{non.cons}$$

$$E_c = E_{c\,OA} + E_{c\,AB} = \frac{1}{2} I_{OA}\,\dot\theta^2 + \frac{1}{2} m v_A^2$$
$$= \frac{1}{6} m L^2 \dot\theta^2 + \frac{1}{2} m L^2 \dot\theta^2 = \frac{2}{3} m L^2 \dot\theta^2$$

$$E_p = E_{p\,ressort\,linéaire} + E_{p\,OA} + E_{p\,AB} + E_{p\,ressort\,spiral}$$
$$= \frac{1}{2} k (l - l_0)^2 + mg\frac{L}{2}\sin\theta + mgL\sin\theta + \frac{1}{2} c\,\theta^2 \qquad l = d - L\sin\theta$$

$$\frac{dE_c}{dt} = \frac{1}{3} m L^2 \dot\theta\ddot\theta + m L^2 \dot\theta\ddot\theta = \frac{4}{3} m L^2 \dot\theta\ddot\theta$$

$$\frac{dE_p}{dt} = k(d - L\sin\theta - l_0)(-L\dot\theta\cos\theta) + \frac{3}{2} mgL\dot\theta\cos\theta + c\,\theta\dot\theta$$

$$\mathcal{P}_{n.c.} = \mathcal{P}_{Ressort\,spiral} = -\chi\dot\theta^2$$

$$\frac{4}{3} m L^2 \cancel{\dot\theta}\ddot\theta + \frac{3}{2} mgL\cancel{\dot\theta}\cos\theta + c\,\theta\cancel{\dot\theta} + k(d - L\sin\theta - l_0)(-L\cancel{\dot\theta}\cos\theta) + \chi\dot\theta^{\cancel{2}} = 0$$

$$\boxed{\frac{4}{3} m L^2 \ddot\theta + \chi\dot\theta + c\,\theta + \left[\frac{3}{2} mgL - kL(d - L\sin\theta - l_0)\right]\cos\theta = 0}$$

faibles amplitudes de mouvements : $\theta = \varepsilon$ $\quad \cos\theta = 1$ $\quad \sin\theta = \varepsilon$
$\dot\theta = \dot\varepsilon$ $\quad \ddot\theta = \ddot\varepsilon$

$$\boxed{\frac{4}{3} m L^2 \ddot\varepsilon + \chi\dot\varepsilon + (kL^2 + c)\varepsilon + \frac{3}{2} mgL - kL(d - l_0) = 0}$$

32) équilibre : $\dot{\theta} = 0$; $\ddot{\theta} = 0 \Rightarrow \dot{\varepsilon} = 0$, $\ddot{\varepsilon} = 0$

$\theta = 0 \rightarrow \varepsilon = 0$

$$\Rightarrow \frac{3}{2} mgL - kL(d - l_0) = 0$$

$$d = \frac{3mgL}{kL} + l_0$$

$$\boxed{d = \frac{3}{2}\frac{mg}{k} + l_0}$$

$$\Rightarrow \boxed{\frac{4}{3} m L^2 \ddot{\varepsilon} + \chi \dot{\varepsilon} + (kL^2 + c)\varepsilon = 0}$$

33) L'équat différentielle se met sous la forme :

$$\ddot{\varepsilon} + 2\xi\omega_0 \dot{\varepsilon} + \omega_0^2 \varepsilon = 0$$

$$\ddot{\varepsilon} + \frac{3}{4}\frac{\chi}{mL^2}\dot{\varepsilon} + \frac{3}{4}\frac{kL^2 + c}{mL^2}\varepsilon = 0$$

avec $\boxed{\omega_0 = \frac{1}{2}\left(3\frac{kL^2 + c}{mL^2}\right)^{1/2}}$

et $2\xi\omega_0 = \frac{3}{4}\frac{\chi}{mL^2} \Rightarrow \boxed{\xi = \frac{3}{8\omega_0}\frac{\chi}{mL^2}}$