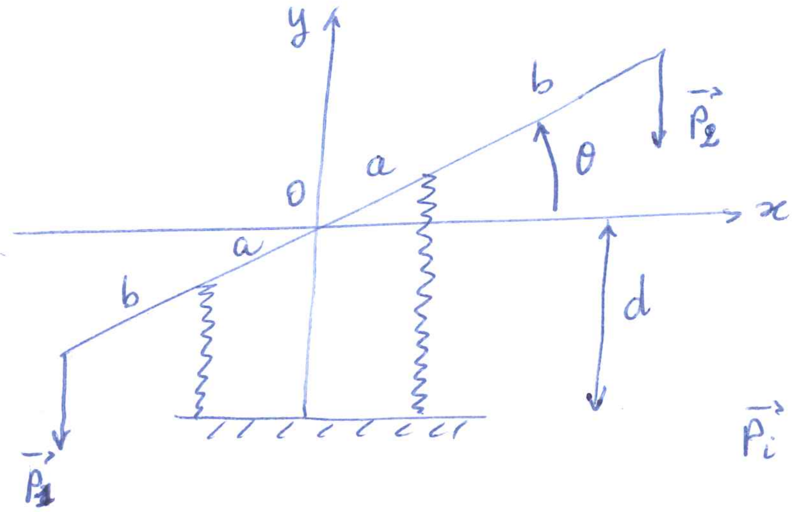
1)



$$\vec{P_i} = - m_i g \, \vec{e_y}$$

$$\vec{T_1} = -k\,(l_1 - l_0)\,\vec{e_y} = -k\,(d - a\sin\theta - l_0)\,\vec{e_y}$$

$$\vec{T_2} = -k\,(l_2 - l_0)\,\vec{e_y} = -k\,(d + a\sin\theta - l_0)\,\vec{e_y}$$

$$\sum \vec{\mathcal{M}_O}(\vec{F_{ext}}) = \vec{0}$$

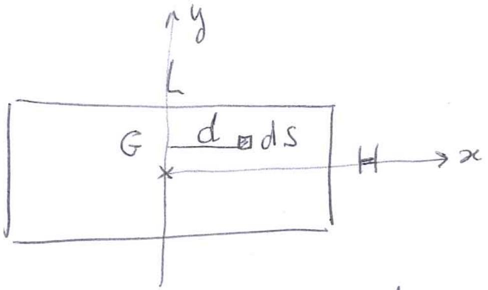
$$\begin{vmatrix} a\cos\theta \\ a\sin\theta \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -k(d + a\sin\theta - l_0) \\ 0 \end{vmatrix} + \begin{vmatrix} -a\cos\theta \\ -a\sin\theta \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -k(d - a\sin\theta - l_0) \\ 0 \end{vmatrix}$$

$$+ \begin{vmatrix} (a+b)\cos\theta \\ (a+b)\sin\theta \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -m_2 g \\ 0 \end{vmatrix} + \begin{vmatrix} -(a+b)\cos\theta \\ -(a+b)\sin\theta \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -m_1 g \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

$$-ka\cancel{\cos\theta}\,(d + a\sin\theta - l_0) + ka\cancel{\cos\theta}\,(d - a\sin\theta - l_0) + (a+b)\cancel{\cos\theta}\,(m_1 - m_2)\,g = 0$$

$$\theta_s = \arcsin\left[\frac{(m_1 - m_2)\,g\,(a+b)}{2a^2k}\right]$$

2) 21)



$$I_{Gy} = \int d^2\,dm \qquad dm = \sigma\,dS = \sigma\,dx\,dy$$

$$d = x$$

$$= \sigma \int x^2\,dx \int dy$$

$$= \sigma \int_{-\frac{L}{2}}^{\frac{L}{2}} x^2\,dx \int_{-\frac{H}{2}}^{\frac{H}{2}} dy = \sigma \left[\frac{x^3}{3}\right]_{-\frac{L}{2}}^{\frac{L}{2}} \left[y\right]_{-\frac{H}{2}}^{\frac{H}{2}}$$

$$\boxed{I_{Gy} = \sigma \frac{L^3}{12} H}$$

22) plaque pleine de dimensions $l \times h$: $I'_{Gy} = \sigma \frac{l^3}{12} h$

$$\Rightarrow \quad I_{Gy} = \frac{\sigma}{12}\left(L^3 H - l^3 h\right)$$

$$m = \sigma S = \sigma\,(LH - lh)$$

$$\Rightarrow \quad \boxed{I_{Gy} = \frac{m}{12} \frac{(L^3 H - l^3 h)}{(LH - lh)}}$$

23)

$$I_{Oy} = I_{Gy} + \left(\frac{L}{2}\right)^2 m$$

$$\boxed{I_{Oy} = \frac{m}{12}\left[\frac{L^3 H - l^3 h}{LH - lh}\right] + \frac{m}{4} L^2}$$

24)

$$I_{Oy} = \frac{20}{12}\left[\frac{0{,}5 - \frac{0{,}4}{8}}{0{,}5 - \frac{0{,}4}{2}}\right] + \frac{20}{4} = \frac{5}{3}\left(\frac{0{,}45}{0{,}3}\right) + 5 = \frac{5}{3}\cdot\frac{3}{2} + 5 = \frac{15}{2}\ \text{kg.m}^2$$

3) 3.1) $$\frac{dE}{dt} = \mathcal{P}_{non.cons}$$

$$E_c = E_{c_{OA}} + E_{c_{AB}} = \frac{1}{2} I_{OA}\, \dot{\theta}^2 + \frac{1}{2} m v_A^2$$

$$= \frac{1}{6} mL^2 \dot{\theta}^2 + \frac{1}{2} m L^2 \dot{\theta}^2 = \frac{2}{3} m L^2 \dot{\theta}^2$$

$$E_p = E_{p_{ressort\ linéaire}} + E_{p_{OA}} + E_{p_{AB}} + E_{p_{ressort\ spiral}}$$

$$= \frac{1}{2} k (l - l_0)^2 + mg\frac{L}{2}\sin\theta + mgL\sin\theta + \frac{1}{2} c\theta^2 \qquad l = d - L\sin\theta$$

$$\frac{dE_c}{dt} = \frac{1}{3} mL^2 \dot{\theta}\ddot{\theta} + mL^2\dot{\theta}\ddot{\theta} = \frac{4}{3} mL^2 \dot{\theta}\ddot{\theta}$$

$$\frac{dE_p}{dt} = k(d - L\sin\theta - l_0)(-L\dot{\theta}\cos\theta) + \frac{3}{2} mgL\dot{\theta}\cos\theta + c\theta\dot{\theta}$$

$$\mathcal{P}_{n.c} = \mathcal{P}_{ressort\ spiral} = -\chi\dot{\theta}^2$$

$$\frac{4}{3} mL^2 \cancel{\dot{\theta}}\ddot{\theta} + \frac{3}{2} mgL\cancel{\dot{\theta}}\cos\theta + c\theta\cancel{\dot{\theta}} + k(d - L\sin\theta - l_0)(-L\cancel{\dot{\theta}}\cos\theta) + \chi\dot{\theta}^{\cancel{2}} = 0$$

$$\boxed{\frac{4}{3} mL^2 \ddot{\theta} + \chi\dot{\theta} + c\theta + \left[\frac{3}{2} mgL - kL(d - L\sin\theta - l_0)\right]\cos\theta = 0}$$

faibles amplitudes de mouvements : $\theta = \varepsilon \quad \cos\theta = 1 \quad \sin\theta = \varepsilon$
$\dot{\theta} = \dot{\varepsilon} \quad \ddot{\theta} = \ddot{\varepsilon}$

$$\boxed{\frac{4}{3} mL^2 \ddot{\varepsilon} + \chi\dot{\varepsilon} + (kL^2 + c)\varepsilon + \frac{3}{2} mgL - kL(d - l_0) = 0}$$

32) équilibre : $\dot{\theta} = 0 \,;\, \ddot{\theta} = 0 \Rightarrow \dot{\varepsilon} = 0 \,;\, \ddot{\varepsilon} = 0$

$\theta = 0 \rightarrow \varepsilon = 0$

$$\Rightarrow \frac{3}{2} mgL - kL(d - l_0) = 0$$

$$d = \frac{3mgL}{kL} + l_0$$

$$\boxed{d = \frac{3}{2}\frac{mg}{k} + l_0}$$

$$\Rightarrow \boxed{\frac{4}{3} m L^2 \ddot{\varepsilon} + \chi \dot{\varepsilon} + (kL^2 + c)\varepsilon = 0}$$

33) L'équat° différentielle se met sous la forme :

$$\ddot{\varepsilon} + 2\xi\omega_0 \dot{\varepsilon} + \omega_0^2 \varepsilon = 0$$

$$\ddot{\varepsilon} + \frac{3}{4}\frac{\chi}{mL^2}\dot{\varepsilon} + \frac{3}{4}\frac{kL^2 + c}{mL^2}\varepsilon = 0$$

avec $\boxed{\omega_0 = \frac{1}{2}\left(3\frac{kL^2 + c}{mL^2}\right)^{1/2}}$

et $2\xi\omega_0 = \frac{3}{4}\frac{\chi}{mL^2} \Rightarrow \boxed{\xi = \frac{3}{8\omega_0}\frac{\chi}{mL^2}}$