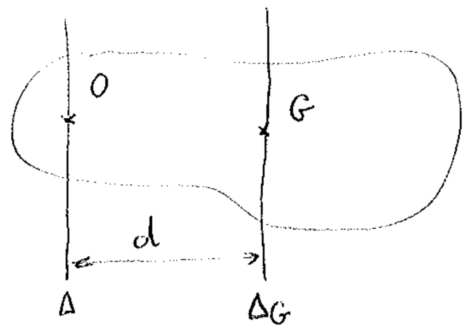
Juin 2017

1) 11)



$\Delta \parallel \Delta_G$

$$I_\Delta = I_{\Delta G} + m d^2$$

12) a) : $E_c = \frac{1}{2} m\, v(G)^2$

b) : $E_c = \frac{1}{2} I_\Delta \omega^2$

c) : $E_c = \frac{1}{2} m\, v(G)^2 + \frac{1}{2} I(G, \vec{u})\, \omega^2$

2) 21) $\vec{R} + (m_1 + m_2 + M_1 + M_2)\vec{g} = \vec{0}$

pas de frottement en O : $\vec{F_i} = F_i\, \vec{e_y}$

$\Rightarrow \quad R - (m_1 + m_2 + M_1 + M_2) g = 0$

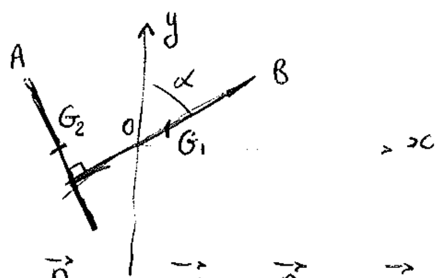
$\Rightarrow \quad R = +(m_1 + m_2 + M_1 + M_2) g$

$\vec{R} = +(m_1 + m_2 + M_1 + M_2)\, g\, \vec{e_y}$

22) $\sum \vec{\mathcal{M}}_O(\vec{F_{ext}}) = \vec{0}$

$\vec{\mathcal{M}}_O(\vec{F_{m_2}}) + \vec{\mathcal{M}}_O(\vec{F_{m_1}}) + \vec{\mathcal{M}}_O(\vec{F_{M_1}}) + \vec{\mathcal{M}}_O(\vec{F_{M_2}}) = \vec{0}$

$\vec{\mathcal{M}}_O(\vec{R}) = \vec{0}$



$$\vec{OA} \wedge \vec{p_2} + \vec{OB} \wedge \vec{p_1} + \vec{OG_2} \wedge \vec{P_2} + \vec{OG_1} \wedge \vec{P_1} = \vec{0}$$

$$\begin{vmatrix} -a\sin\alpha - 3b\cos\alpha \\ -a\cos\alpha + 3b\sin\alpha \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -m_2 g \\ 0 \end{vmatrix} + \begin{vmatrix} 3a\sin\alpha \\ 3a\cos\alpha \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -m_1 g \\ 0 \end{vmatrix} +$$

$$\begin{vmatrix} -a\sin\alpha - b\cos\alpha \\ -a\cos\alpha + b\sin\alpha \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -M_2 g \\ 0 \end{vmatrix} + \begin{vmatrix} a\sin\alpha \\ a\cos\alpha \\ 0 \end{vmatrix} \wedge \begin{vmatrix} 0 \\ -M_1 g \\ 0 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

$$\rightarrow \quad + m_2 \cancel{g} \left(a\sin\alpha + 3b\cos\alpha\right) - 3a\sin\alpha \; m_1 \cancel{g}$$

$$+ M_2 \cancel{g} \left(a\sin\alpha + b\cos\alpha\right) - M_1 \cancel{g}\, a\sin\alpha = 0$$

$$\sin\alpha\left(m_2 a - 3m_1 a + M_2 a - M_1 a\right) + \cos\alpha\left(3b m_2 + b M_2\right) = 0$$

$$a\tan\alpha\left(m_2 - 3m_1 + M_2 - M_1\right) + b\left(3m_2 + M_2\right) = 0$$

$$\boxed{\tan\alpha = -\frac{b}{a}\left(\frac{3m_2 + M_2}{m_2 - 3m_1 + M_2 - M_1}\right)}$$

3) On peut prendre I_{Oz} :

$$\left.\begin{aligned} d(M, Oz) &= r\sin\theta \\ dm &= \rho\, r^2 \sin\theta \, d\theta \, d\varphi \, dr \end{aligned}\right\} \; I_{Oz} = \rho \int_{R_1}^{R_2}\int_0^{\pi}\int_0^{2\pi} (r\sin\theta)^2 r^2 \sin\theta \, d\theta \, dr \, d\varphi$$

$$I_{Oz} = \rho \underbrace{\int_{R_1}^{R_2} r^4 dr}_{\frac{R_2^5 - R_1^5}{5}} \; \underbrace{\int_0^{\pi} \sin^3\theta \, d\theta}_{\underbrace{(2-2/3)}_{4/3}} \; \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} = \rho \frac{\left(R_2^5 - R_1^5\right)}{15} \cdot 8\pi$$

$$m = \frac{4}{3}\pi\left(R_2^3 - R_1^3\right)\rho$$

$$\rightarrow \quad I_{Oz} = \frac{2}{5} m \left(\frac{R_2^5 - R_1^5}{R_2^3 - R_1^3}\right)$$

4) voir TDs