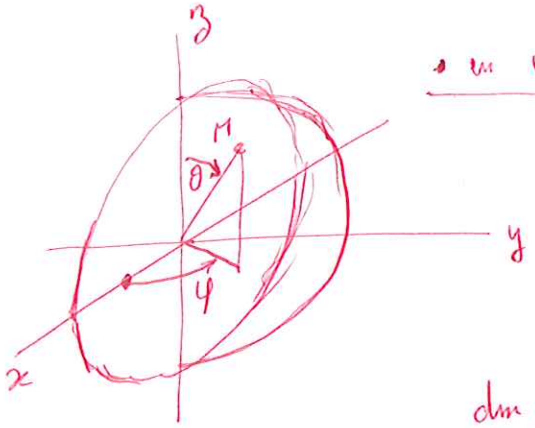
Centre de Masse $\frac{1}{2}$ sphère pleine



• en utilisant dV en coordonnées sphérique

Oy : axe de symétrie : $\begin{cases} x_G = 0 \\ z_G = 0 \end{cases}$

$$m\,y_G = \int y\,dm$$

$$dm = \rho\,dV = \rho\,r^2 dr \sin\theta\,d\theta\,d\varphi$$

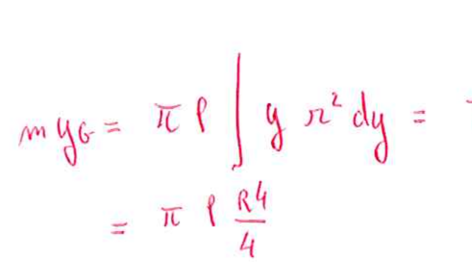
$$y = r\sin\theta\sin\varphi$$

$$\rightarrow \quad m\,y_G = \rho \int_0^R r^3 dr \int_0^\pi \sin^2\theta\,d\theta \int_0^{2\pi} \sin\varphi\,d\varphi$$

$$= \rho\,\frac{R^4}{4} \cdot \frac{1}{2}\left[\pi - 0\right] \cdot 2 = \rho\pi\frac{R^4}{4}$$

$$\left(\sin^2\theta = \frac{1-\cos 2\theta}{2}\right)$$

avec $m = \rho \cdot \frac{2}{3}\pi R^3$ $\quad \rightarrow \quad \boxed{y_G = \frac{3R}{8}}$



• en découpant en tranches élémentaires

pour chaque tranche : $y_{Gi} = y$

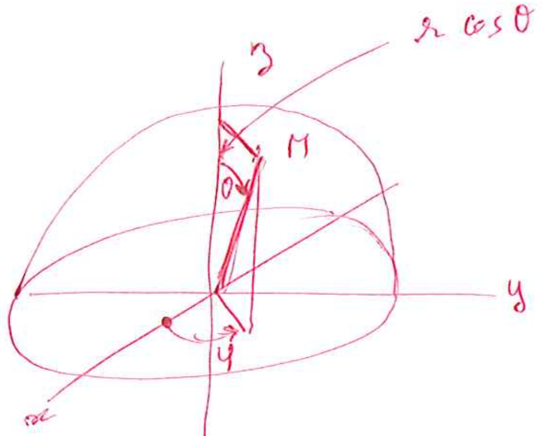
$m\,y_G = \int y_{Gi}\,dm_i$ avec $\begin{cases} dm_i = \rho\pi r^2 dy \\ r^2 = R^2 - y^2 \end{cases}$

$$m\,y_G = \pi\rho \int y\,r^2 dy = \pi\rho \int_0^R y\left(R^2 - y^2\right) dy = \pi\rho R^2 \cdot \frac{R^2}{2} - \pi\rho\frac{R^4}{4}$$

$$= \pi\rho\frac{R^4}{4}$$

$m = \rho \cdot \frac{2}{3}\pi R^3 \Rightarrow$ $\quad \boxed{y_G = \frac{3R}{8}}$

On pouvait aussi utiliser :



$z = r\cos\theta$

$dV = r^2 dr \sin\theta \, d\theta \, d\varphi$

$$m\,\vec{OG} = \int z\,dm = \rho \int z\,dV$$

$$= \rho \int_0^R r^3 dr \int_0^{\frac{\pi}{2}} \sin\theta\cos\theta\, d\theta \int_0^{2\pi} d\varphi$$

$$= \rho \frac{R^4}{4} \int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{2} d\theta \,.\, 2\pi$$

$$= \rho \frac{R^4}{4} \cdot 2\pi \cdot \frac{1}{2}\left[\frac{-1}{2}\cos 2\theta\right]_0^{\frac{\pi}{2}} = \frac{\rho R^4 \pi}{2} \cdot \frac{1}{2}\left[\frac{1}{2} + \frac{1}{2}\right] = \rho\pi \frac{R^4}{4}$$

$$m = \rho \frac{4}{3}\pi R^3 \quad \rightarrow \quad \boxed{z_G = \frac{3R}{8}}$$

Moment d'inertie d'1 sphère creuse

% à un diamètre

sphère pleine

$$I_{Oz} = \int d^2\, dm \qquad d(M, Oz) = r \sin\theta$$

$$dm = \rho\, r^2 \sin\theta\, d\theta\, dr\, d\varphi$$

$$I_{Oz} = \rho \int_0^R \int_0^\pi \int_0^{2\pi} (r \sin\theta)^2 r^2 \sin\theta\, d\theta\, dr\, d\varphi$$

$$= \rho \underbrace{\int_0^R r^4\, dr}_{\frac{R^5}{5}} \int_0^\pi \sin^3\theta\, d\theta \underbrace{\int_0^{2\pi} d\varphi}_{2\pi}$$

$$\int_0^\pi \sin^3\theta\, d\theta = \int_0^\pi \sin\theta\,(1 - \cos^2\theta)\, d\theta = \underbrace{\int_0^\pi \sin\theta\, d\theta}_{2} - \int_0^\pi \sin\theta \cos^2\theta\, d\theta$$

$$\int_0^\pi \overbrace{\sin\theta \underbrace{\cos^2\theta}_{u^n} d\theta}^{-du} = \frac{1}{3}\left[-\cos^3\theta\right]_0^\pi = \frac{2}{3}$$

$$\rightarrow I_{Oz} = \frac{8}{15} \rho \pi R^5 \qquad m = \frac{4}{3}\pi R^3 \rho \quad \rightarrow \boxed{I_{Oz} = \frac{2}{5} m R^2}$$

sphère creuse

$$I_{Oz} = \rho \int_{R_1}^{R_2} r^4 dr \int_0^\pi \sin^3\theta\, d\theta \int_0^{2\pi} d\varphi$$

$$= \rho\, \frac{R_2^5 - R_1^5}{5} \cdot \frac{4}{3} \cdot 2\pi = \frac{8}{15} \rho \pi \left(R_2^5 - R_1^5\right)$$

$$m = \frac{4}{3} \pi \rho \left(R_2^3 - R_1^3\right)$$

$$\rightarrow \boxed{I_{Oz} = \frac{2}{5} m \left(\frac{R_2^5 - R_1^5}{R_2^3 - R_1^3}\right)}$$

Roulement d'1 sphère creuse sur 1 plan incliné

Pas de force non-conservative : $\frac{d(E_c+E_p)}{dt} = 0$

déplacement selon Ox

$$E_c = \frac{m \dot{x}_G^2}{2} + \frac{I_{Gz} \dot{\varphi}^2}{2} = \frac{m \dot{x}^2}{2} + \frac{I_{Gz} \dot{x}^2}{2 R_2^2} \qquad (\dot{x} = -R_2 \dot{\varphi})$$

$$E_p = m g \Delta h = -m g x \sin \alpha$$

Δh : hauteur parcourue pour 1 distance x.

$$\frac{d(E_c+E_p)}{dt} = m \dot{x}\ddot{x} + \frac{I_{Gz}}{R_2^2} \dot{x}\ddot{x} - m g \dot{x} \sin \alpha = 0$$

$$\rightarrow \boxed{\ddot{x} = \frac{m g \sin \alpha}{m + \frac{I_{Gz}}{R_2^2}}}$$

$$\rightarrow \dot{x} = \int \ddot{x} dt = \frac{m g \sin \alpha}{m + \frac{I_{Gz}}{R_2^2}} t + cte$$

C.I : $t=0$, $\dot{x}=0 \Rightarrow cte = 0$

$$\rightarrow x = \int \dot{x} dt = \frac{m g \sin \alpha}{m + \frac{I_{Gz}}{R_2^2}} \frac{t^2}{2} + cte$$

C.I. : $t=0$, $x=0 \rightarrow cte = 0$

A.N. : $\ddot{x} = \frac{m g \sin \alpha}{m + \frac{I_{Gz}}{R_2^2}} = 0{,}081 \ m\,s^{-2}$

$\dot{x} = 0{,}81 \ m\,s^{-1}$

$x = \cancel{\ldots} \ 4{,}05 \ m$