

Quantitative Texture Analysis

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Outline

Qualitative aspects of crystallographic textures

Grains, Crystallites and Crystallographic planes

Normal diffraction

Effects on diffraction diagrams, their limitations

θ - 2θ scans

Asymmetric scans

ω -scans (rocking curves)

Representations of texture: pole figures

Pole Sphere

Stereographic projection

Equal-area projection: Lambert/Schmidt projection

Pole figures

Localisation of crystallographic directions from pole figures

Direct and normalised pole figures

Normalisation

Incompleteness and corrections of pole figures

Single texture component

Multiple texture components

Pole figures and (hkl) multiplicity

A real example

Pole figure types

- Random texture

- Planar textures

- Fibre textures

- Three-dimensional texture

Pole Figures and Orientation spaces

- Mathematical expression of diffraction pole figures and ODF

- From pole figures to the ODF

- Orientations g and pole figures

- Euler angle conventions

- From $f(g)$ to pole figures

- Deal with ODF in the \mathcal{G} space

- Plotting the ODF

Inverse pole figures

ODF refinement

- Generalised spherical harmonics

- WIMV

- Entropy modified WIMV and Entropy maximisation

- ADC, Vector and component methods

- ODF coverage

- Reliability and texture strength estimators

Magnetic QTA

Why needing QTA !!

- correcting texture effects
 - powder XRD
 - spectroscopic methods (P-EXAFS, ESR, Raman ...)
- mollusc phylogeny, fossils
- predicting texture effect on macroscopic anisotropic properties
 - average to get macroscopic tensors
 - simulating elasticity, electric polarisation
 - Bulk Acoustic Waves
 - anisotropic magnetisation
- correlation texture - macroscopic anisotropic properties
 - Thermoelectric Power Factor
 - Pyroelectric coefficients
 - Tauc gap
 - Jc in superconductors
 - Levitation forces and trapped flux

But why classical QTA vanishes

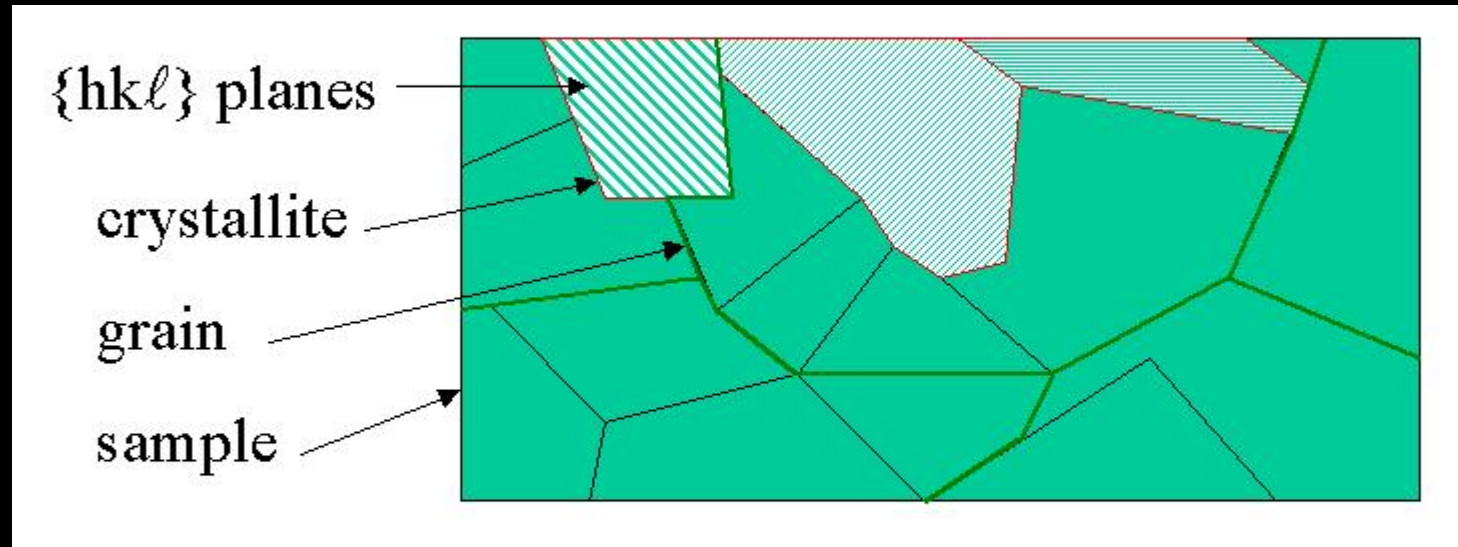
Why needing Combined analysis

Minimum experimental requirements

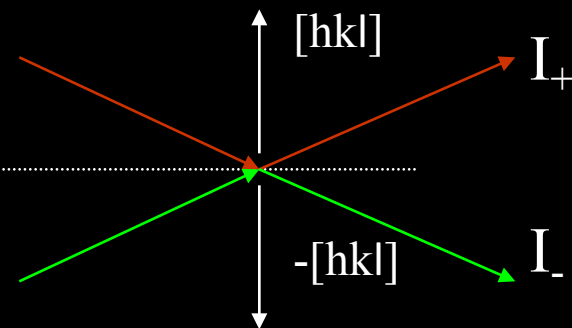
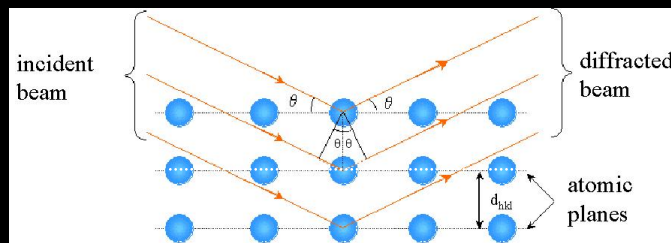
Qualitative aspects of texture

- Polycrystal: aggregate of grains, different phases, sizes, shapes, orientations, stress state, crystallinity, faults ...
- Diffraction:
 - probes lattice planes: crystallites, not grains
 - x-rays, neutrons or electrons
- SEM:
 - grains, not crystallites (coherent, single crystal domains)
 - shape vs crystallographic texture (EBSD)

Grains, crystallites, crystallographic planes

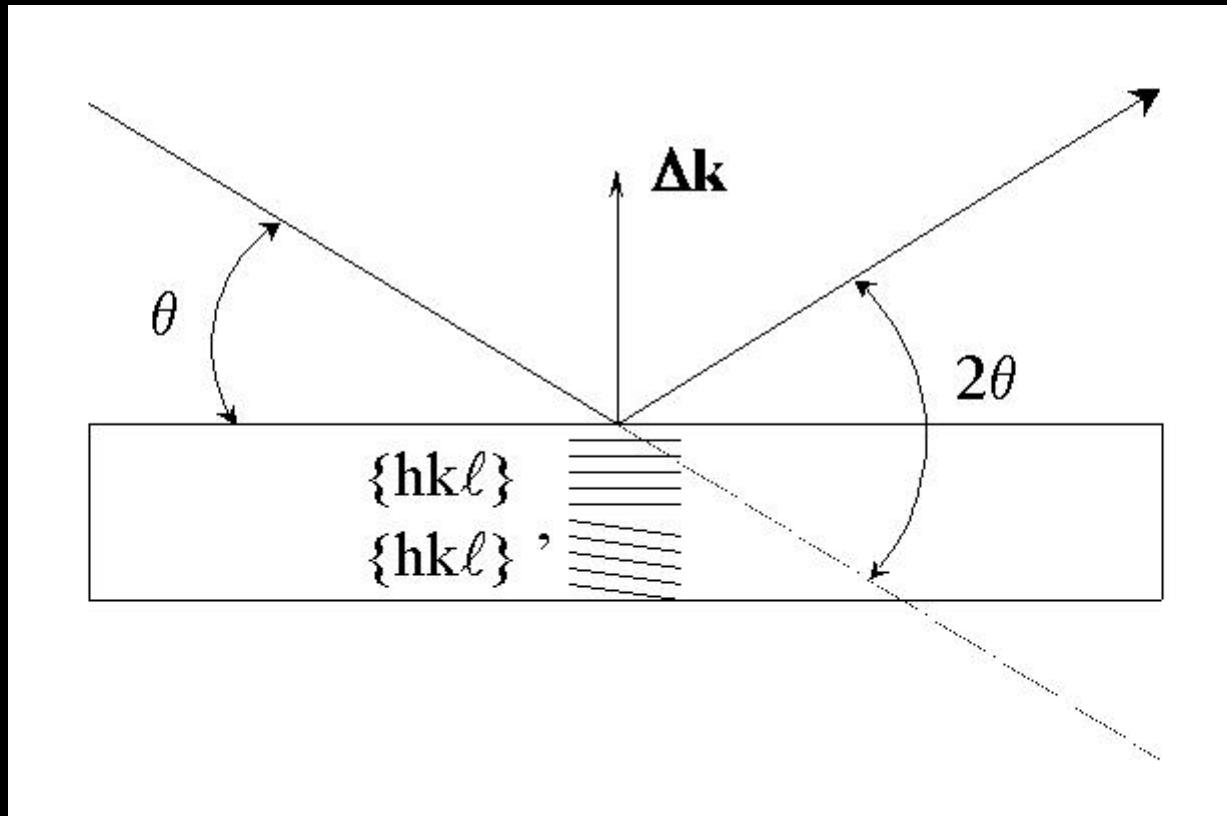


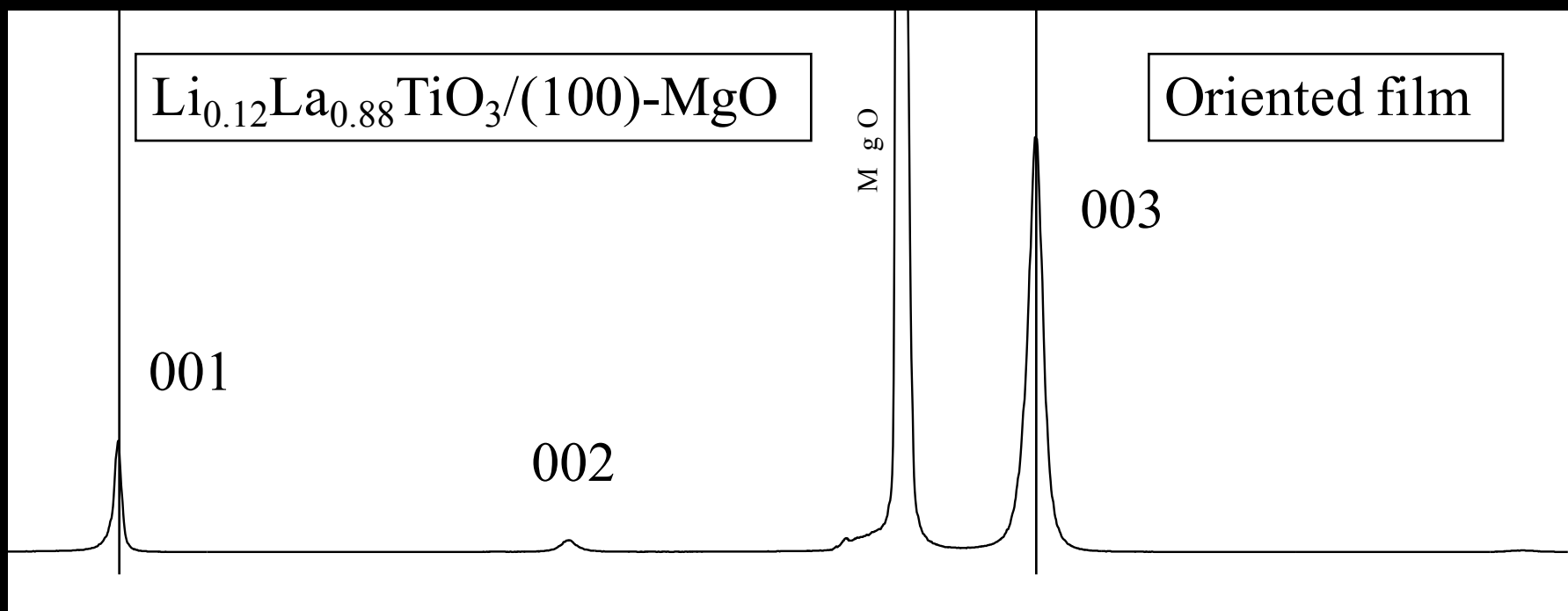
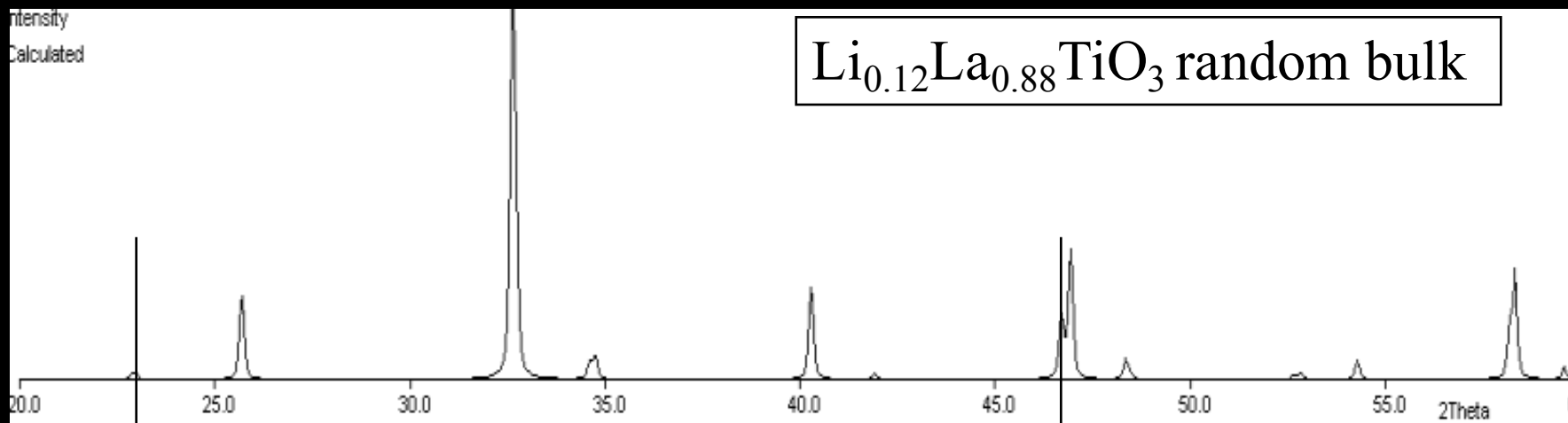
Friedel's law: $I_{hkl} = I_{-h-k-l}$ using normal diffraction
+ or - directions not distinguished



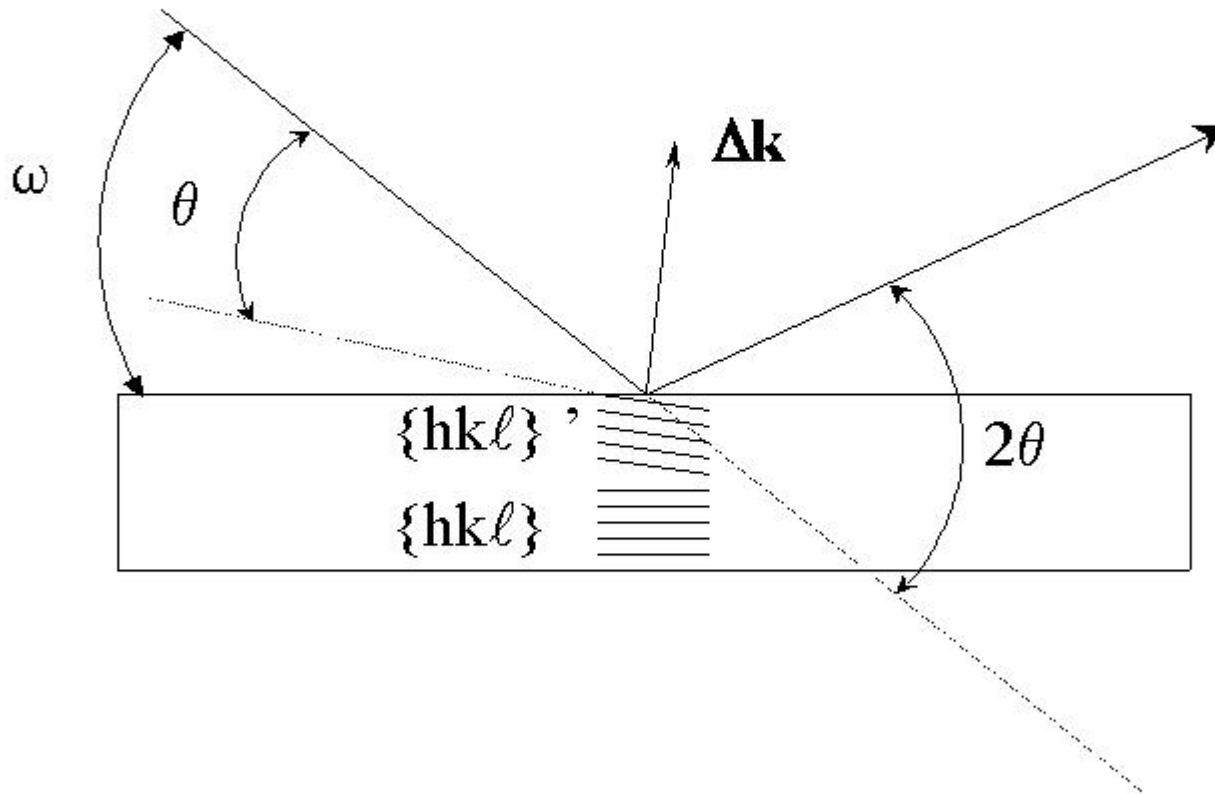
Texture effects on diffraction diagrams

θ - 2θ scan: probes only parallel planes

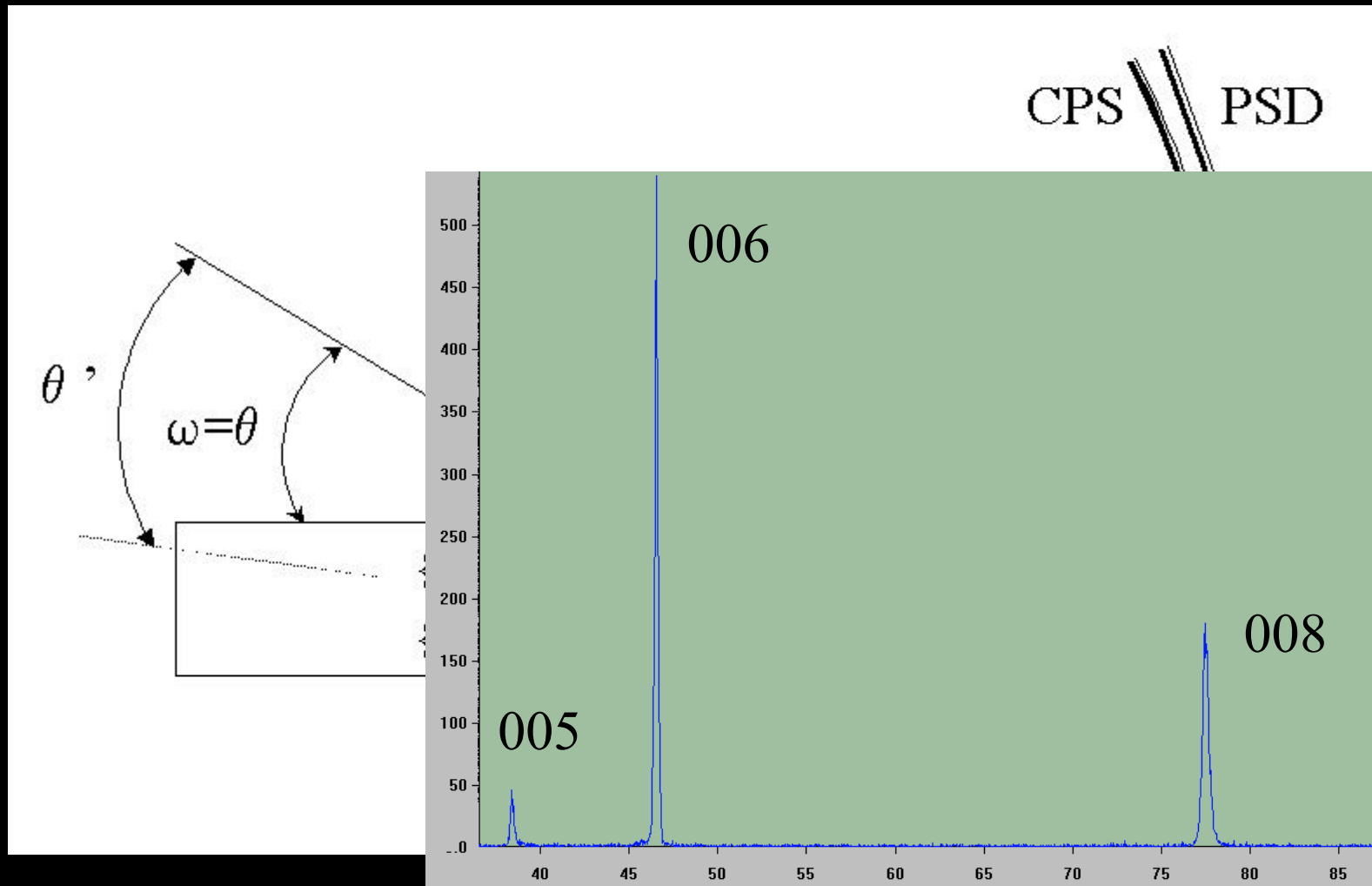




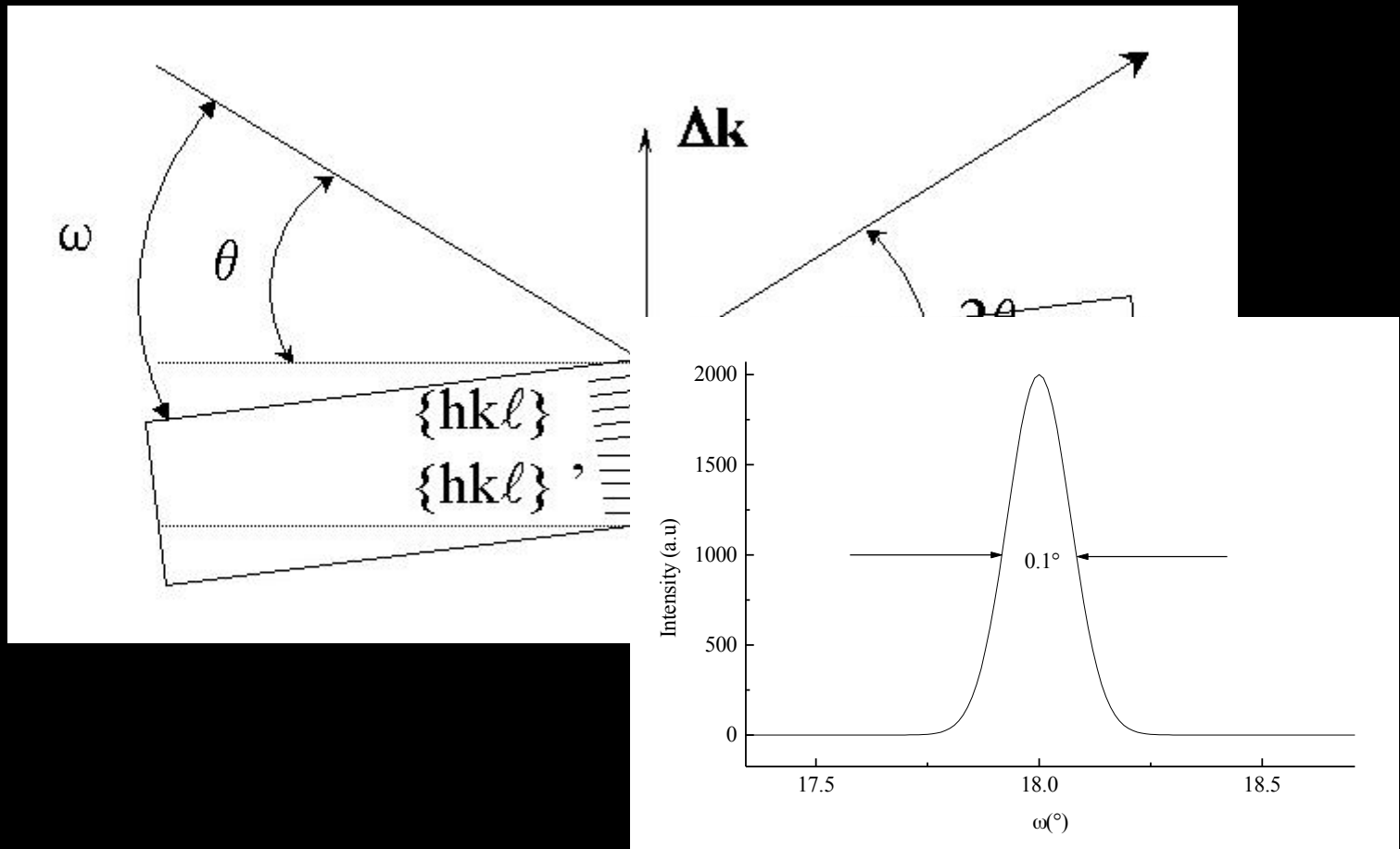
asymmetric scan: probes only inclined planes



mixed scan: probes specific planes for
specific orientations

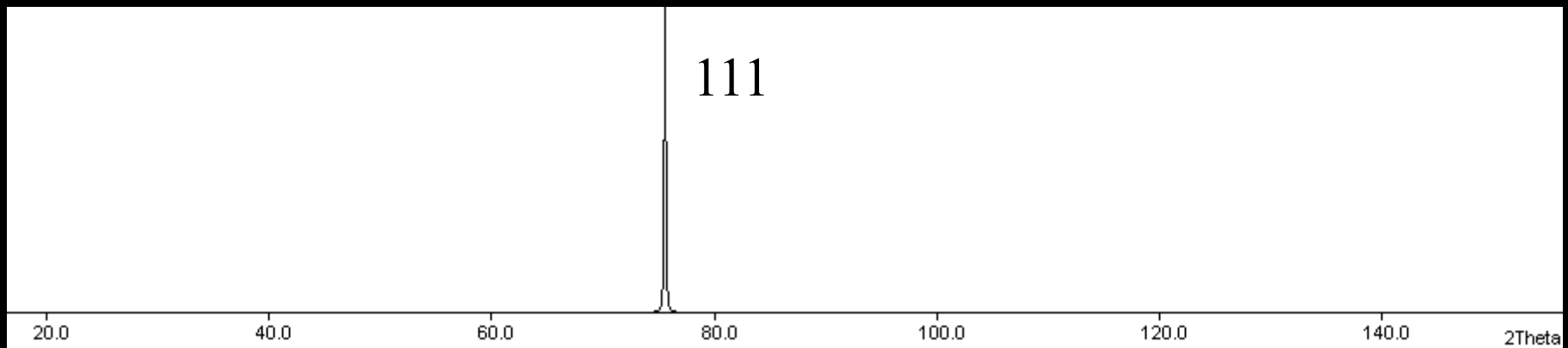


ω scan: probes orientation of only one plane type (fixed θ), only for small ω - θ



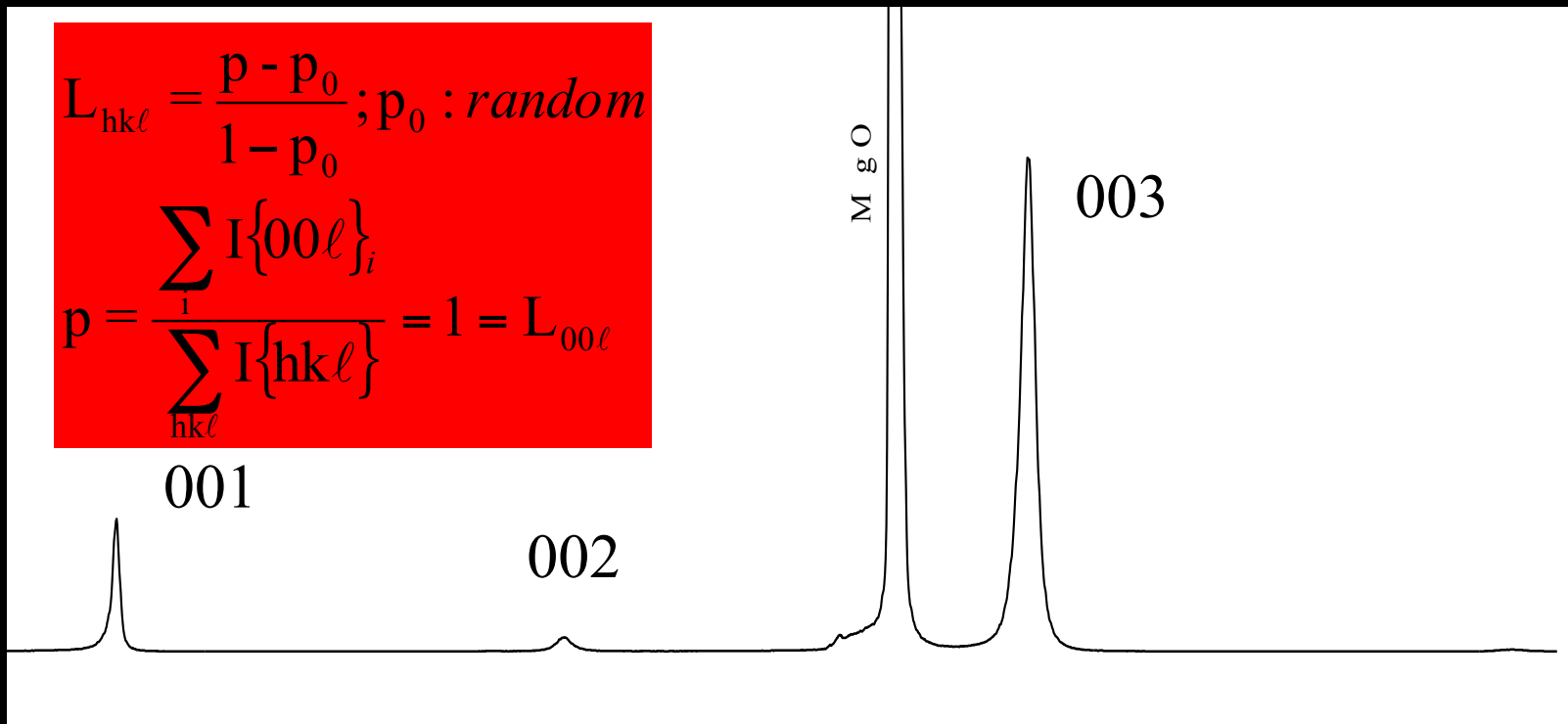
limitations: available θ (or other) range

diamond (Fd3m), 2.52 Å neutrons, up to $2\theta = 150^\circ$

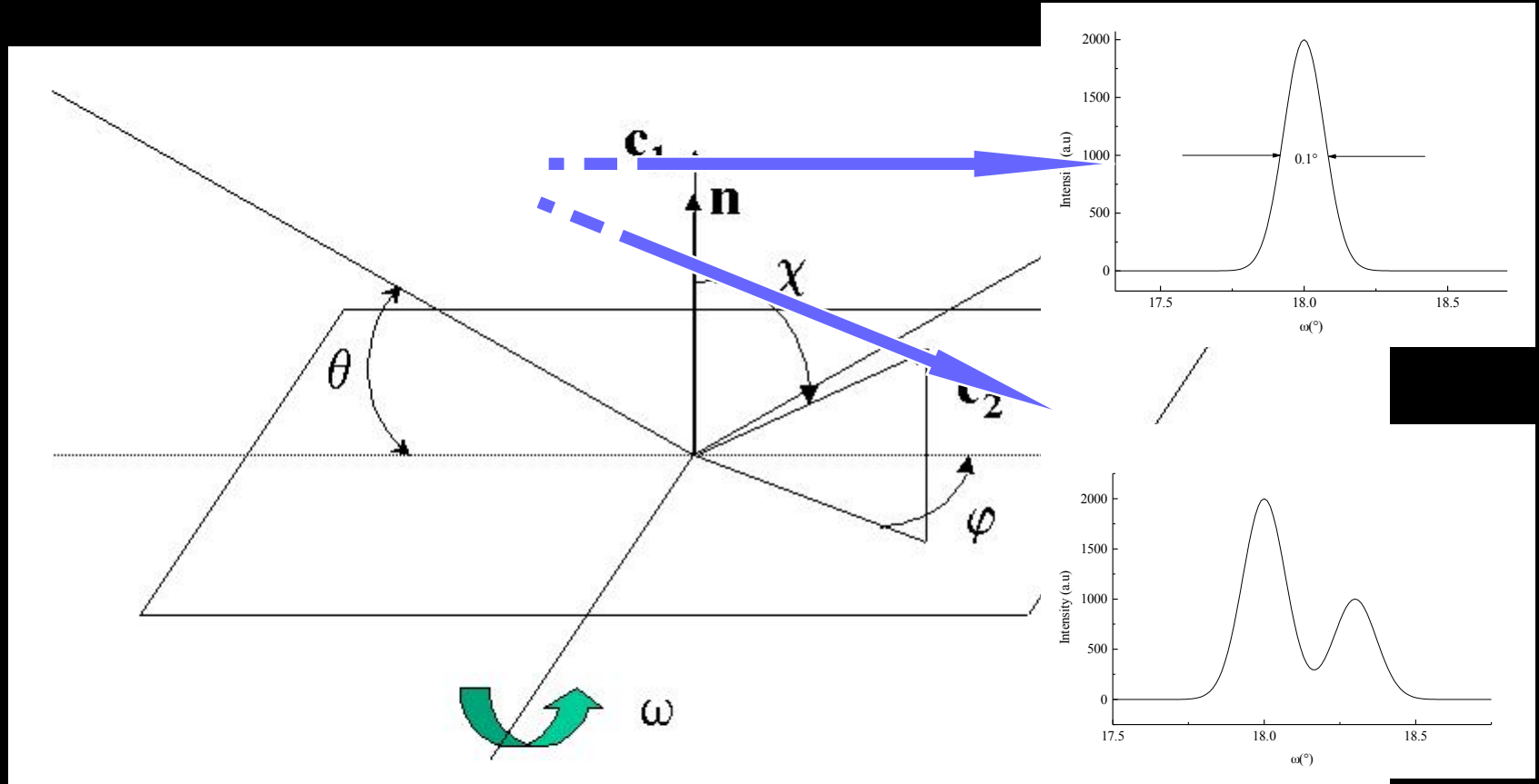


limitations: 2 texture components

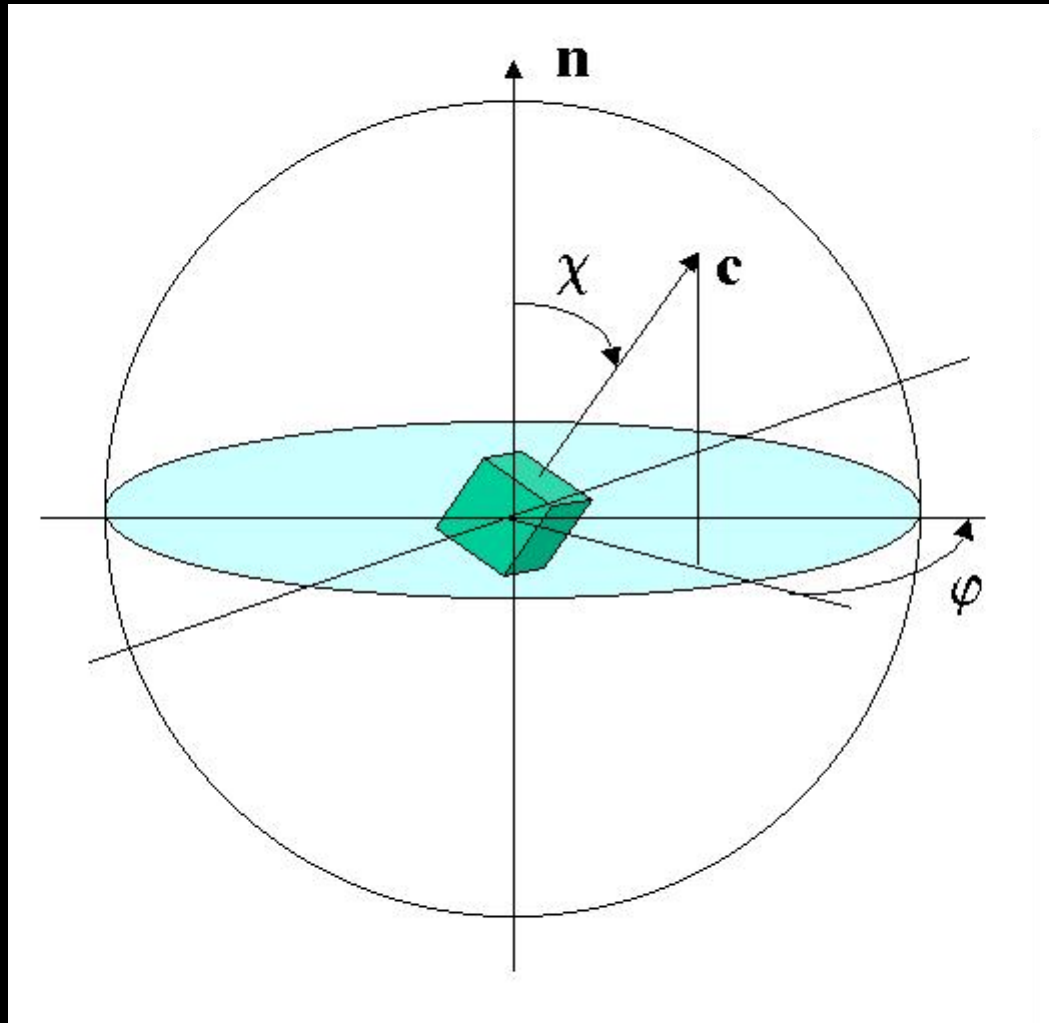
same c-axes direction, but not same a-axes orientation



limitations: 2 texture components, one inclined



Representations of texture: pole figures



One crystallite oriented
in the Pole sphere:

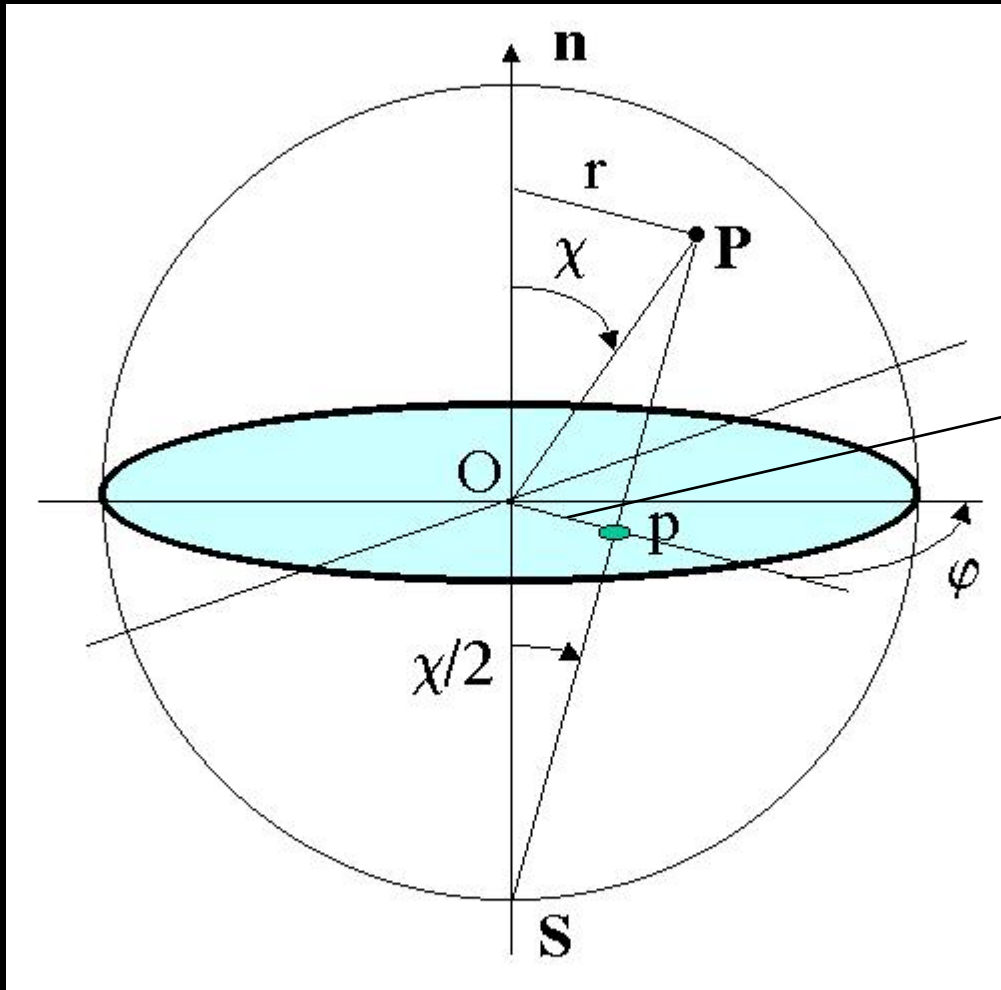
- location of all
 $[hkl] \in$ unit sphere

- $dS = \sin\chi \, d\chi \, d\phi$

- (χ, ϕ) : angles in the
diffractometer space S

Hard to visualise: needs
pole figures

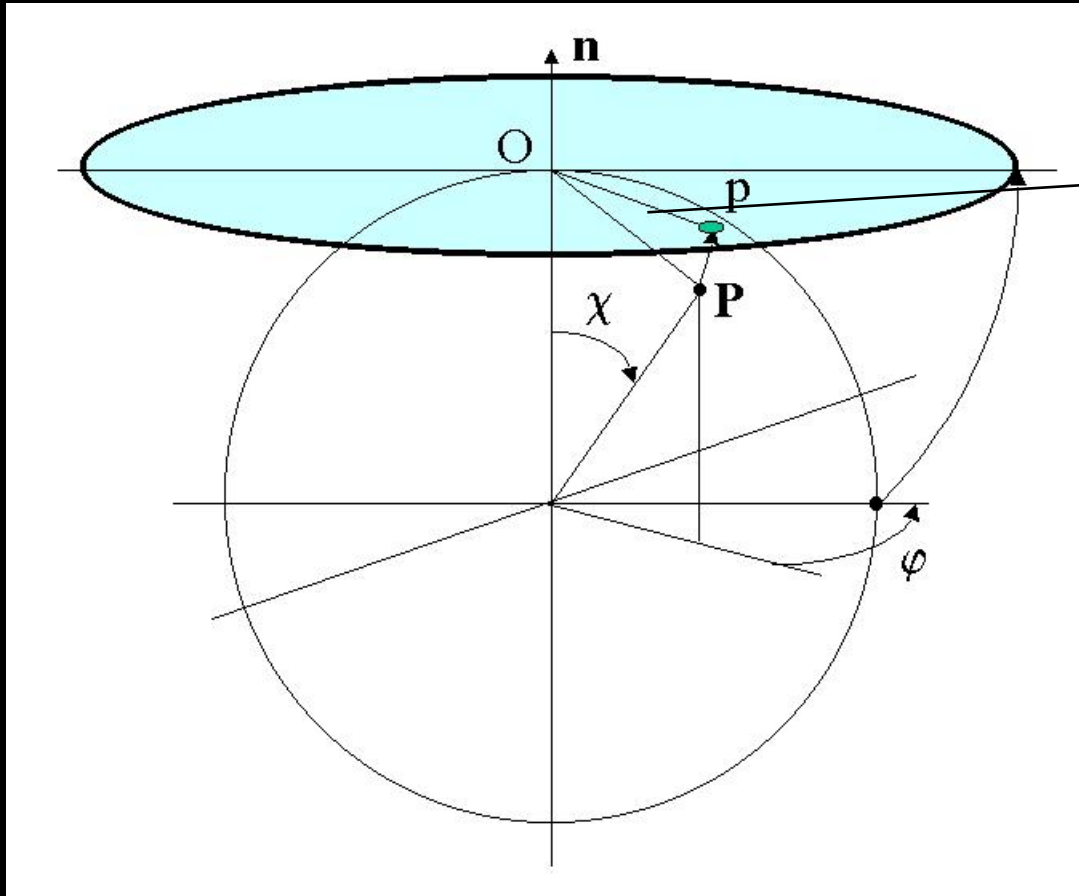
Stereographic projections: equal angle



Poles: $p(r', \phi)$:

$$r' = R \tan(\chi/2)$$

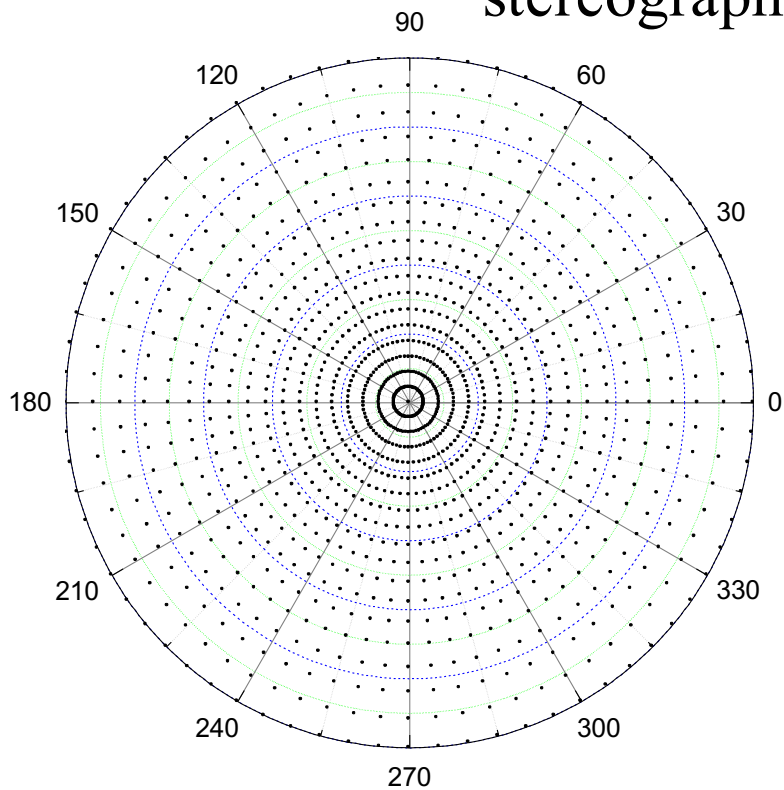
Lambert projections (equal area)



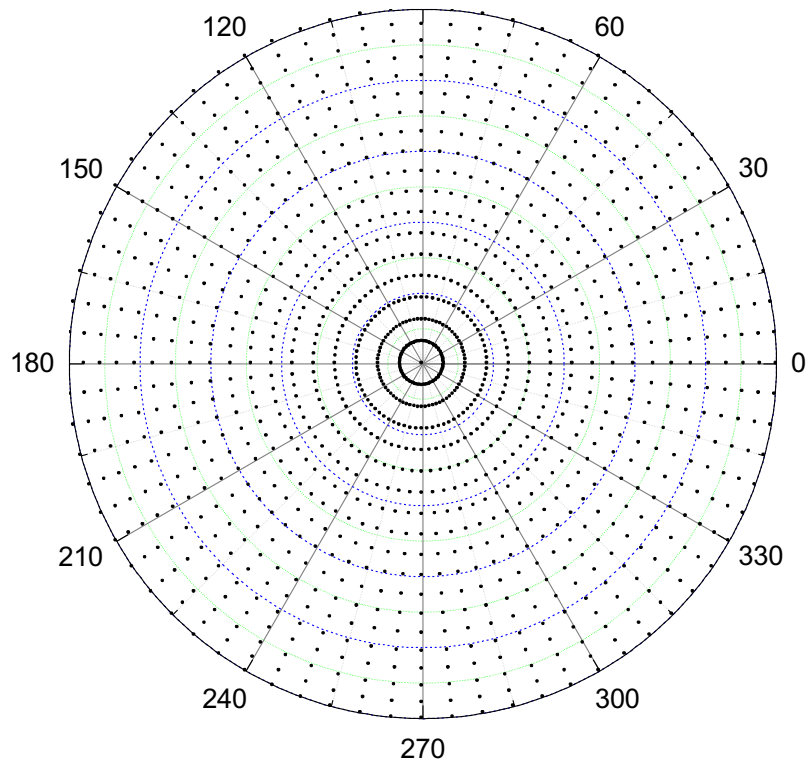
Poles: $p(r', \varphi)$:

$$r' = 2R \sin(\chi/2)$$

stereographic



Lambert/Schmidt

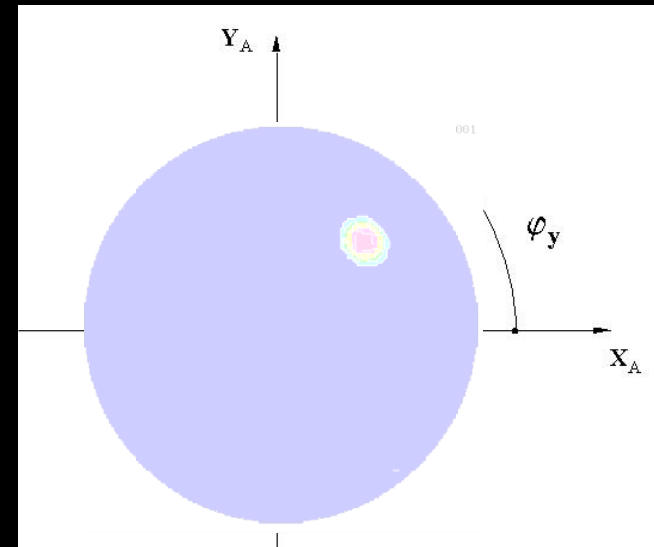
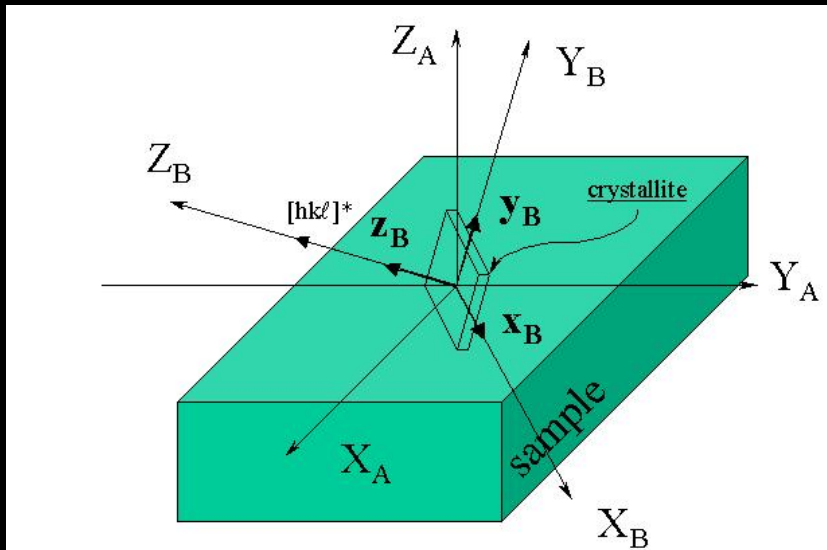


5° x 5° grid: 1368 points

Pole figures

{hkl}-Pole figure: location of distribution densities, for the {hkl} diffracting plane, defined in the crystallite frame K_B , relative to the sample frame K_A .

Pole figures space: Y , with $\mathbf{y} = (\vartheta_y, \varphi_y) = [hkl]^*$

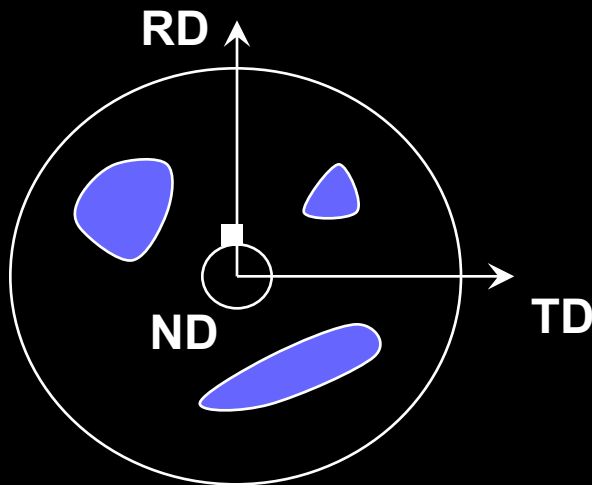


Direct Pole Figure: built on diffracted intensities $I_h(\mathbf{y})$, $\mathbf{h} = \langle hkl \rangle^*$

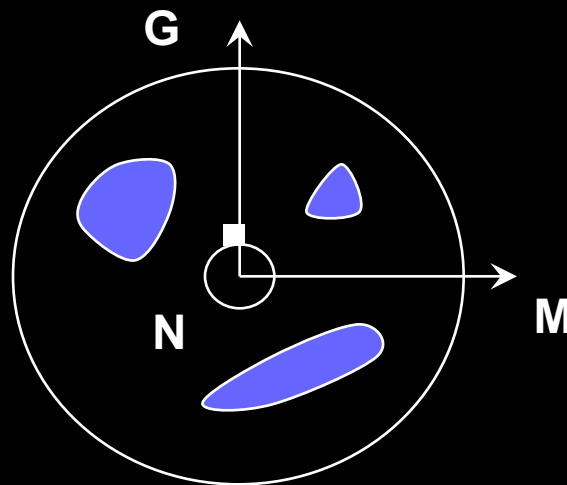
Normalised Pole Figure: built on distribution densities $P_h(\mathbf{y})$

Density unit: the "multiple of a random distribution", or "m.r.d."

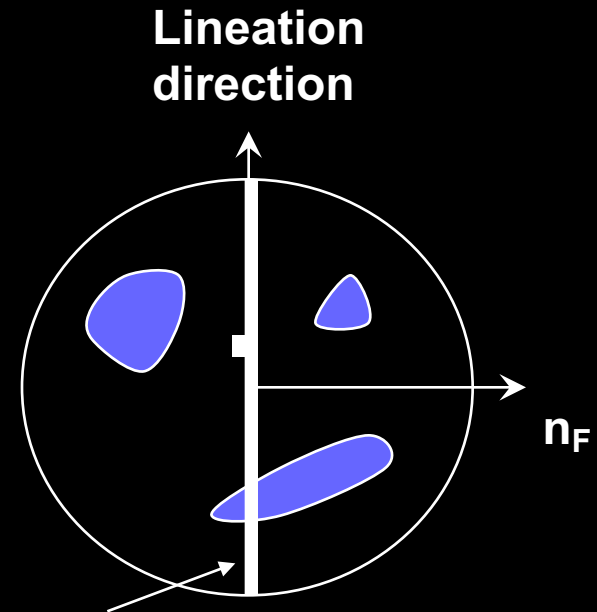
Usual pole figure frames K_A



metallurgy



malacology

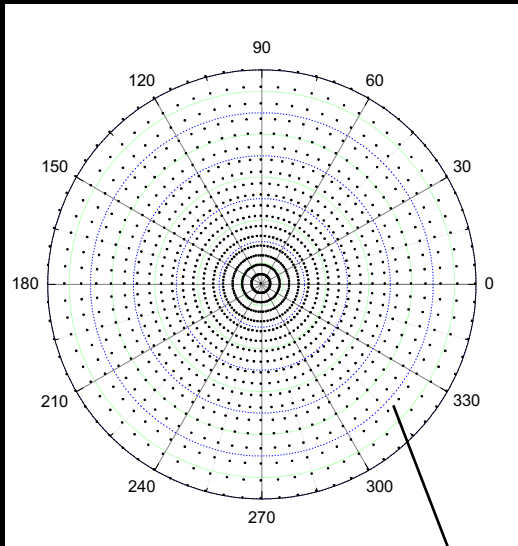


geophysics

Thin films: substrate directions ...

X_A, Y_A, Z_A

Normalisation



$$I_h(\vartheta_y, \varphi_y)$$

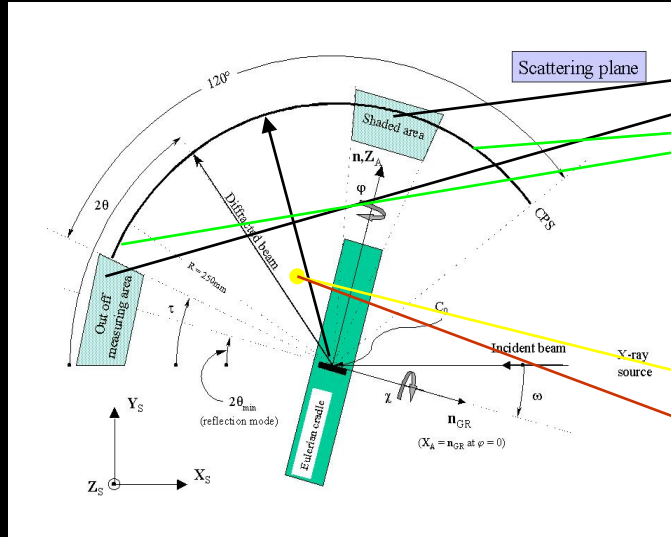
$$I_h^{\text{total}} = \int_{\varphi_y=0}^{2\pi} \int_{\vartheta_y=0}^{\pi/2} I_h(\vartheta_y, \varphi_y) \sin \vartheta_y \, d\vartheta_y \, d\varphi_y$$

$$I_h^{\text{random}} = I_h^{\text{total}} / \int_{\varphi_y=0}^{2\pi} \int_{\vartheta_y=0}^{\pi/2} \sin \vartheta_y \, d\vartheta_y \, d\varphi_y$$

$$P_h(\mathbf{y}) = \frac{I_h(\mathbf{y})}{I_h^{\text{random}}}$$

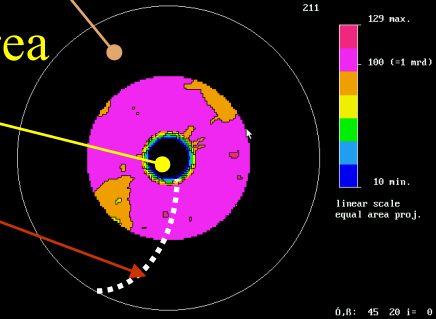
- Only valid for complete pole figures:
neutrons in symmetric geometry
- Needs a refinement strategy to get I^{random} for all h 's

Incompleteness and corrections of pole figures

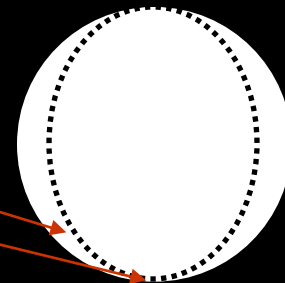
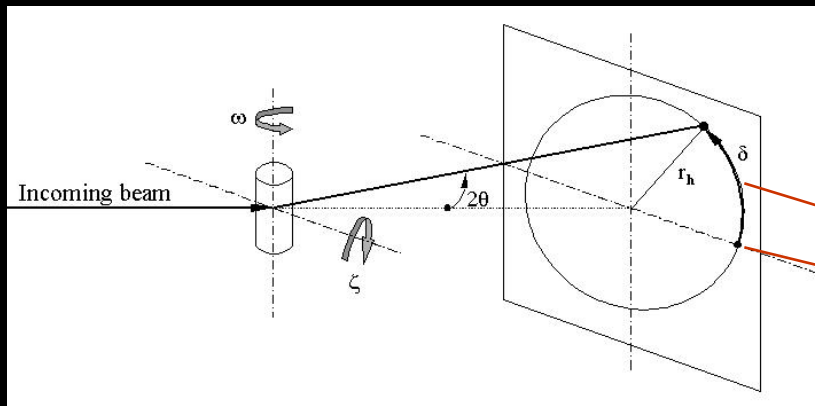


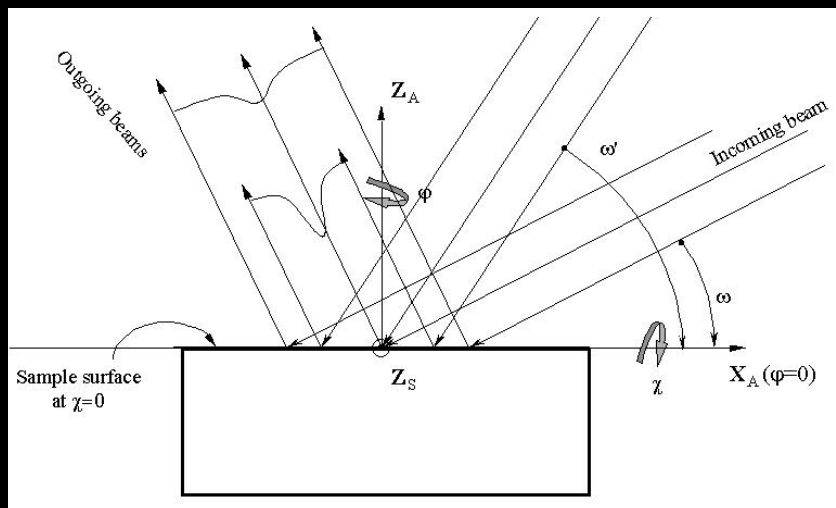
Missing Bragg peaks
 Absorption + volume
 Defocusing (x-rays)

Blind area

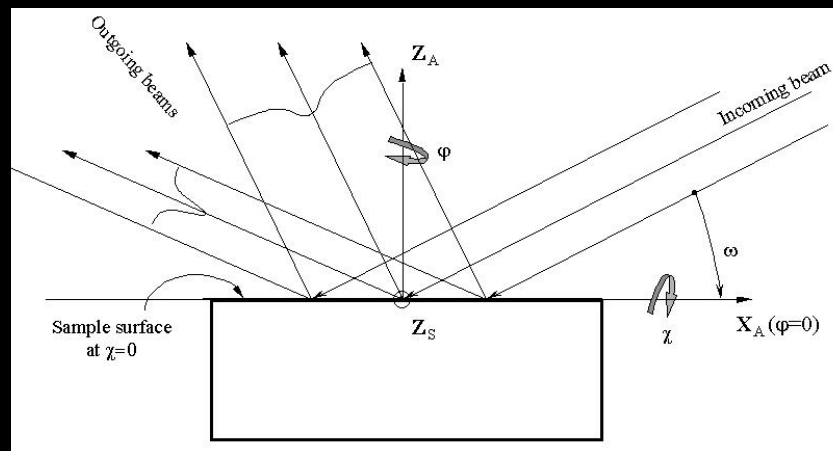


Localisation

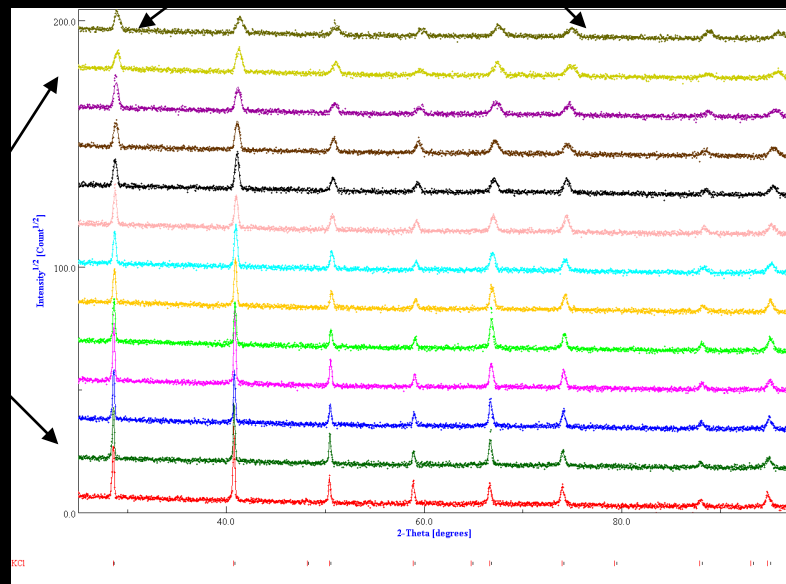
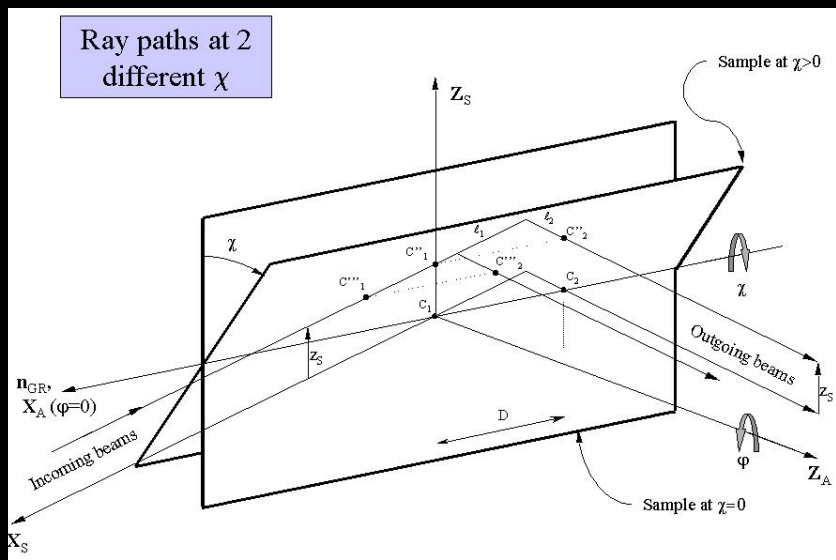




ω -defocusing
 χ -defocusing



2θ -defocusing

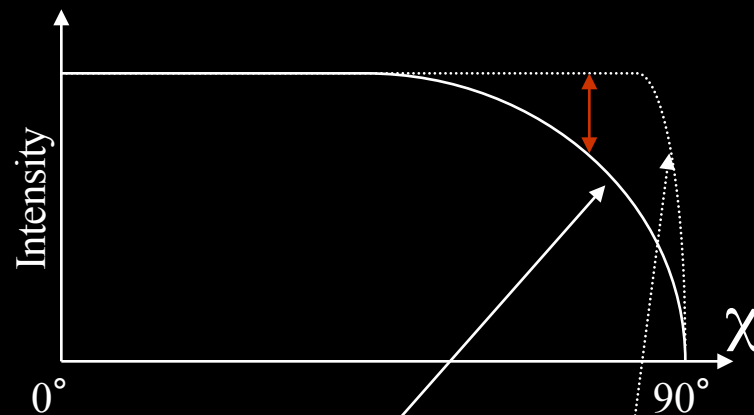


Defocusing corrections:

- Calibration on a random powder

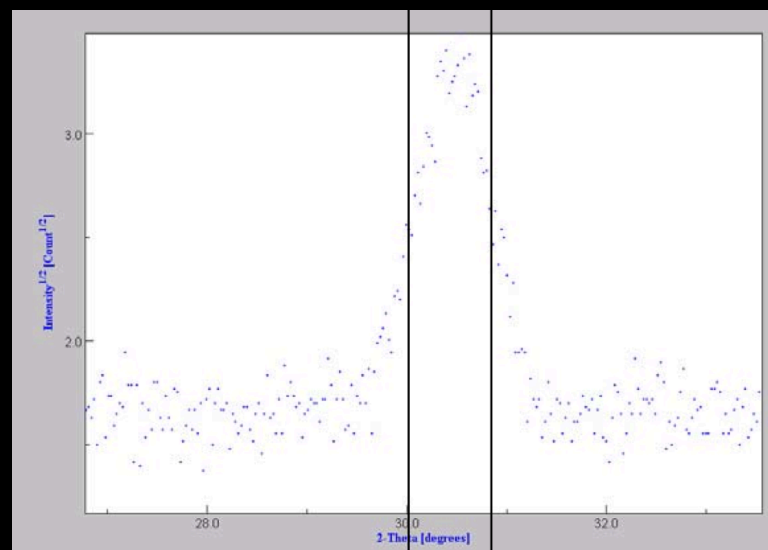
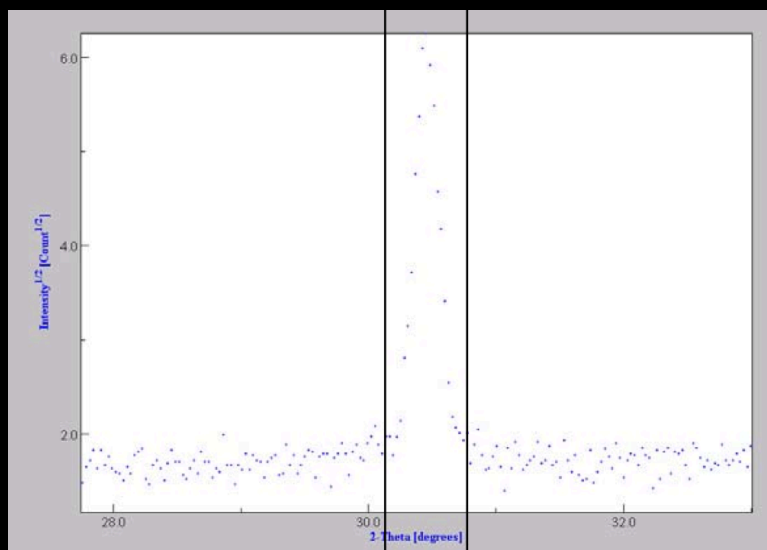
$$I_{\chi,\varphi,\omega,\theta}^{\text{cor}} = I_{\chi,\varphi,\omega,\theta}^{\text{meas}} \frac{I_{0,\omega,\theta}^{\text{rand}}}{I_{\chi,\omega,\theta}^{\text{rand}}} \quad \text{Net intensities (point detector)}$$

$$= \left[I_{\chi,\varphi,\omega,\theta}^{\text{meas}} - I_{0,\omega,\theta}^{\text{bkg}} \frac{I_{\chi,\omega,\theta}^{\text{b}}}{I_{0,\omega,\theta}^{\text{bkg}}} \right] \frac{I_{0,\omega,\theta}^{\text{rand}} - I_{0,\omega,\theta}^{\text{bkg}}}{I_{\chi,\omega,\theta}^{\text{rand}} - I_{0,\omega,\theta}^{\text{bkg}}}$$

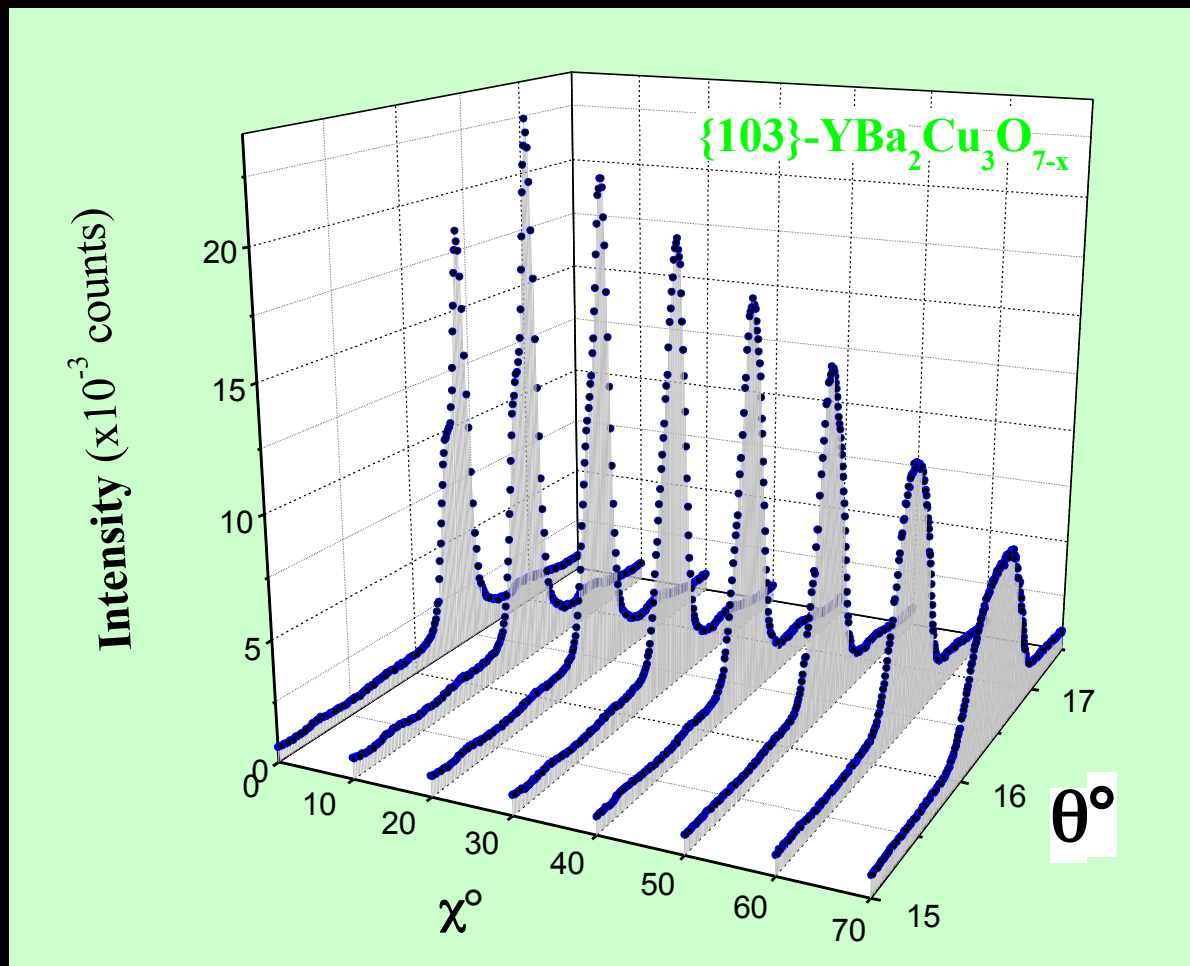


Peak maximum (point detector)
Integrated intensity (1D or 2D detector)

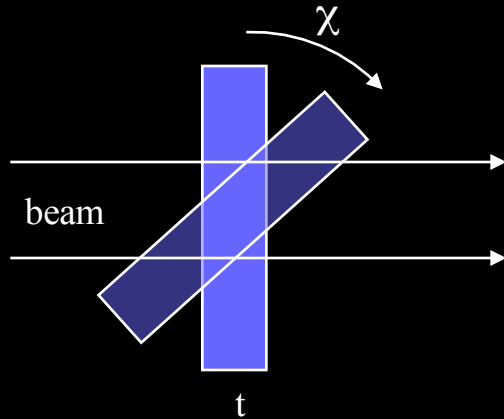
- Total integration of the peak (direct integration or fit)



Overlaps enhance the problems !



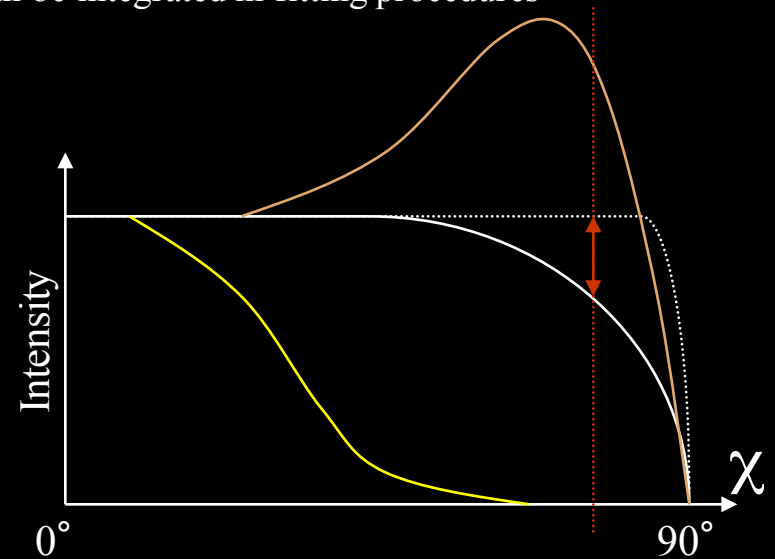
Absorption/Volume corrections:



Specific to each instrumental geometry
 Sample dependent (films, multilayers ...)
 Modifies the defocusing curves
 Can be integrated in fitting procedures

Top film

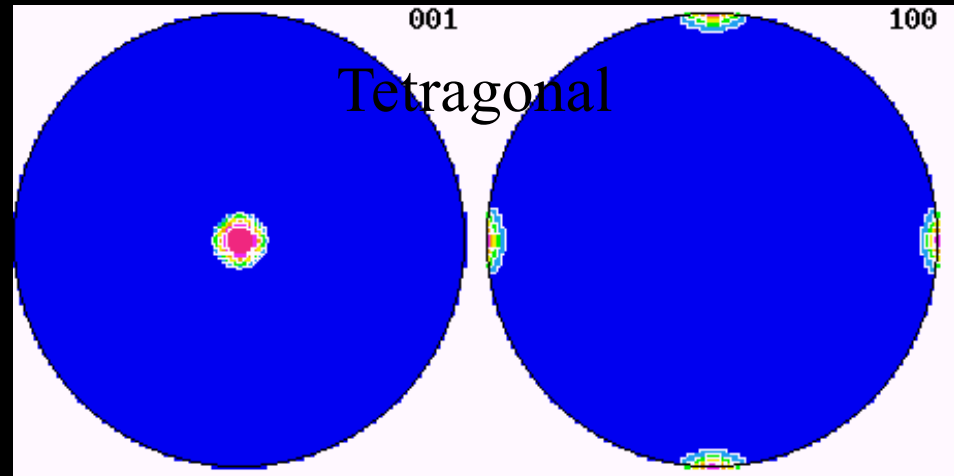
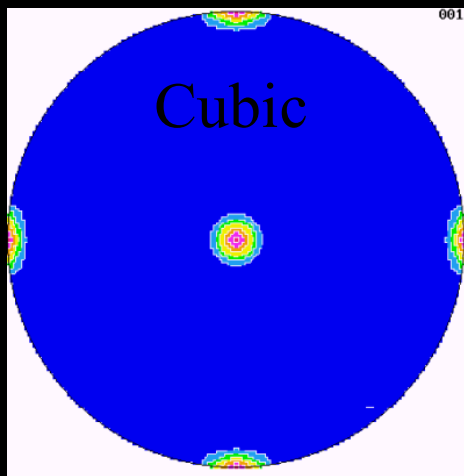
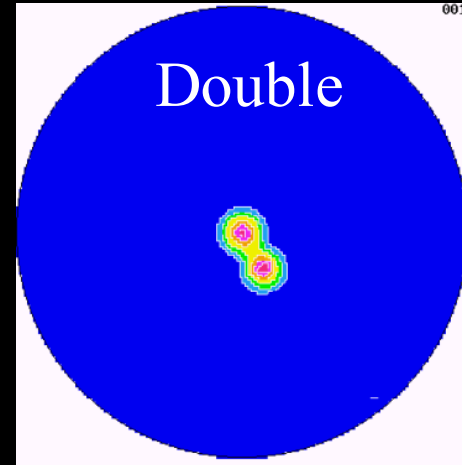
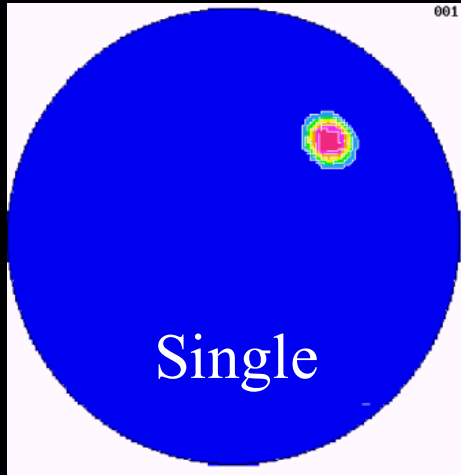
$$I(0) = I(\chi) \frac{(1 - \exp(-2\mu T / \sin \theta_i))}{(1 - \exp(-2\mu T / \sin \theta_i \cos \chi))}$$



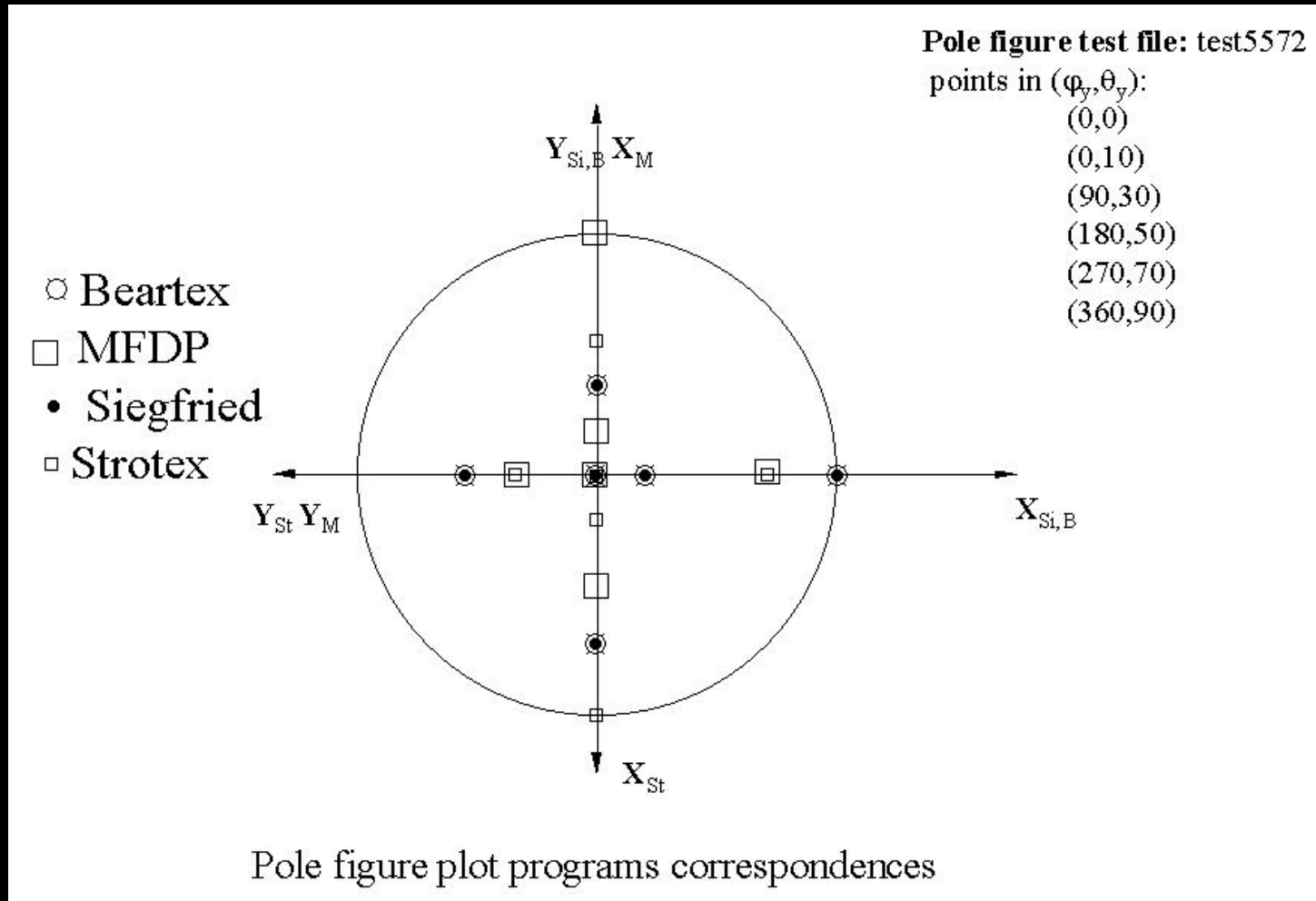
Covered layer

$$I(0) = I(\chi) \frac{(1 - \exp(-2\mu T / \sin \theta_i)) \exp\left(\frac{-2 \sum_j \mu_j T_j}{\sin \theta_i}\right)}{(1 - \exp(-2\mu T / \sin \theta_i \cos \chi)) \exp\left(\frac{-2 \sum_j \mu_j T_j}{\sin \theta_i \cos \chi}\right)}$$

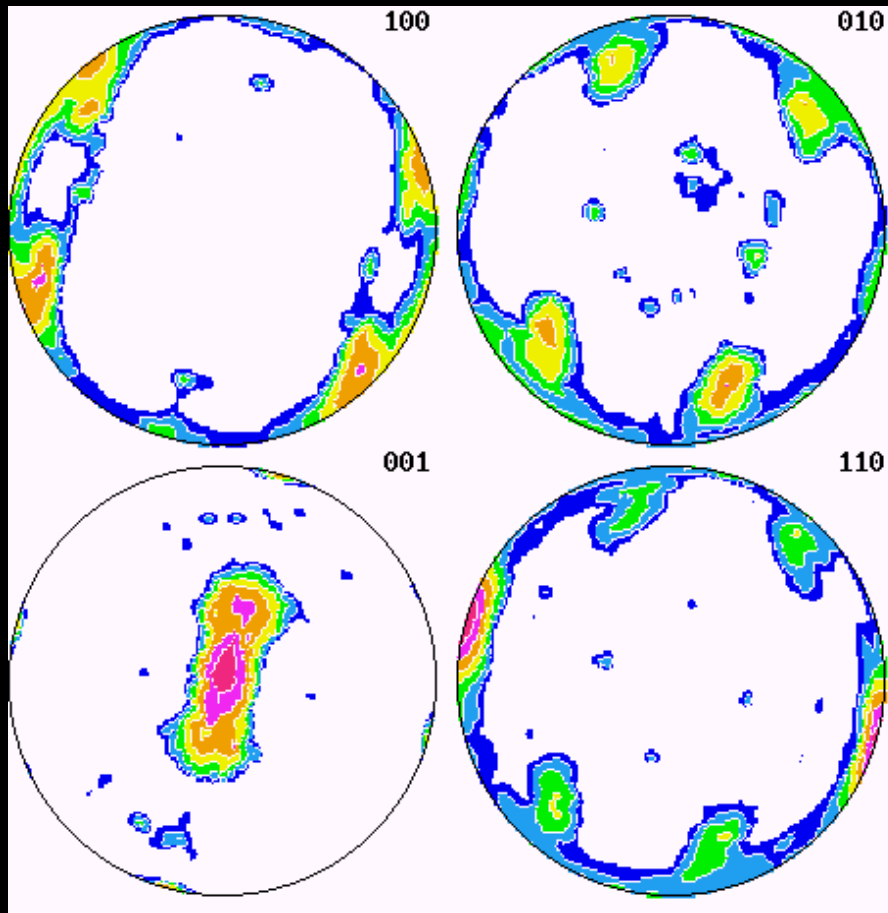
Single or multiple texture components, multiplicity



Program convention ! orientations are ... oriented objects



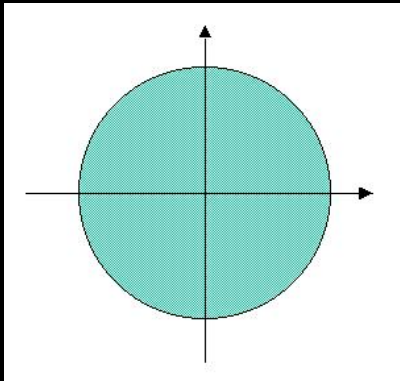
A real example



Cypraea testudinaria

Outer aragonite layer
Pnma space group

Texture types

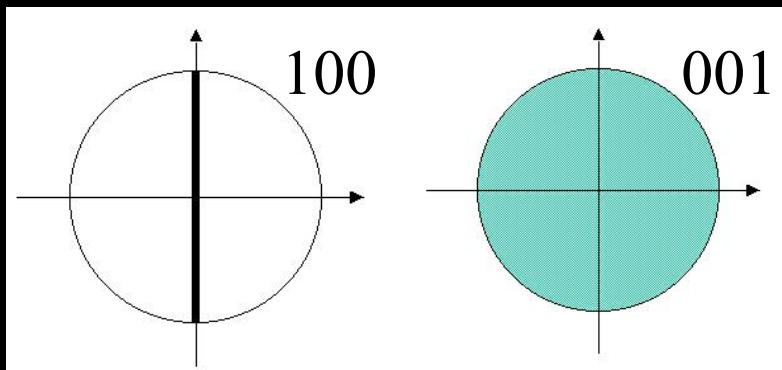


Random texture

3 degrees of freedom

All $P_h(\mathbf{y})$ homogeneous

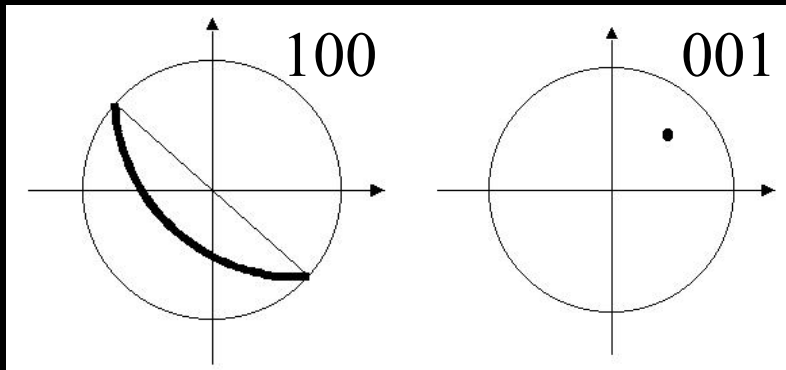
1 m.r.d. density whatever \mathbf{y}



Planar texture

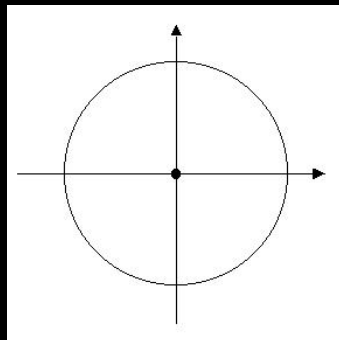
2 degrees of freedom

1 $[hkl]$ at random in a plane



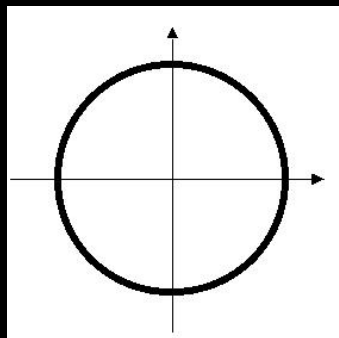
Fibre texture

1 degree of freedom
 1 [hkl] along 1 y direction



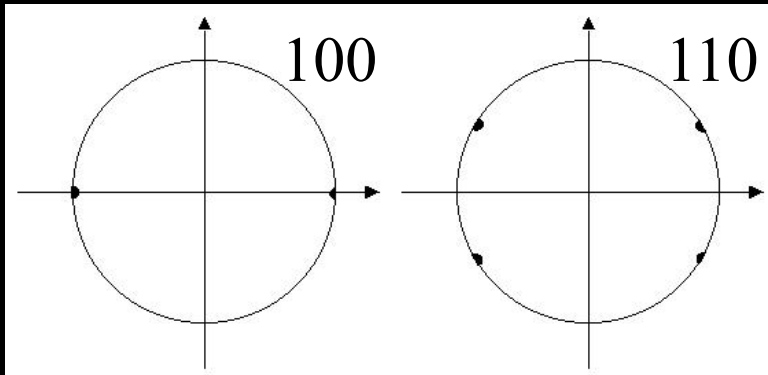
Cyclic-Fibre texture

$$\mathbf{c} // Z_A$$



Cyclic-Planar texture

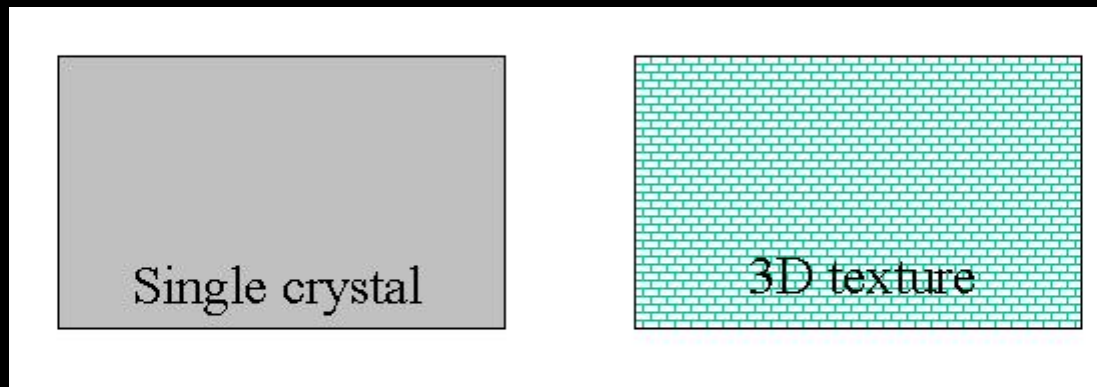
$$\mathbf{c} // (X_A, Y_A)$$



Single crystal-like texture

0 degree of freedom

2 [hkl]'s along 2 y directions



Single-crystal and perfect 3D orientation not distinguished

Pole figure and Orientation spaces

Pole figure expression:

$$\frac{dV(\mathbf{y})}{V} = \frac{1}{4\pi} P_h(\mathbf{y}) d\mathbf{y}$$

$$d\mathbf{y} = \sin\vartheta_y d\vartheta_y d\varphi_y$$

$$\int_{\varphi_y=0}^{2\pi} \int_{\vartheta_y=0}^{\pi/2} P_h(\vartheta_y, \varphi_y) \sin\vartheta_y d\vartheta_y d\varphi_y = 4\pi$$

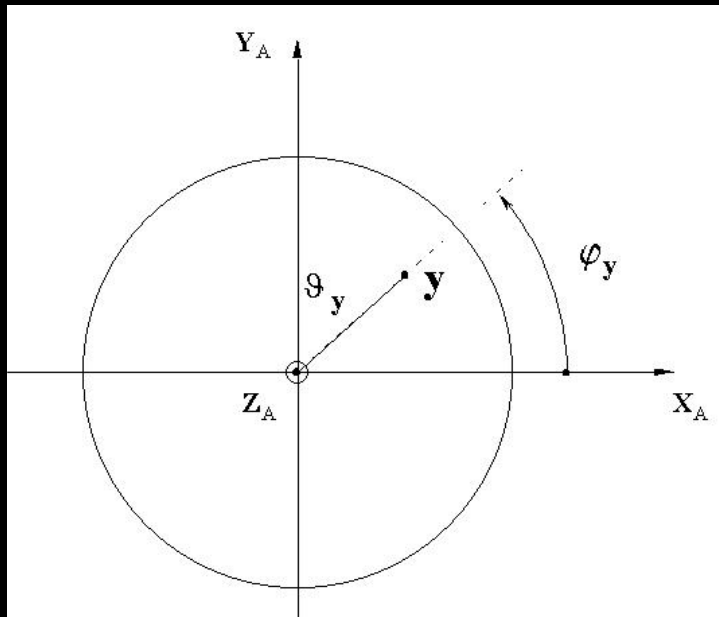
Orientation Distribution Function $f(\mathbf{g})$:

$$\frac{dV(\mathbf{g})}{V} = \frac{1}{8\pi^2} f(\mathbf{g}) d\mathbf{g}$$

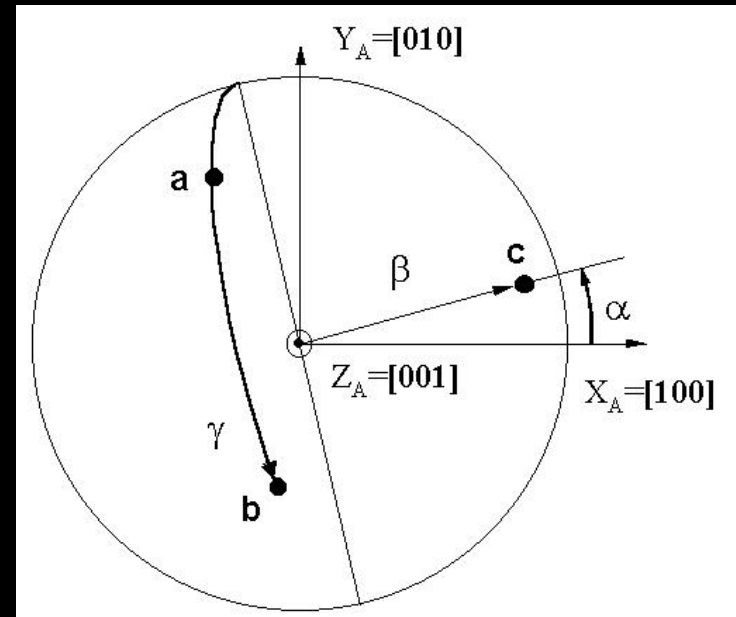
$$d\mathbf{g} = \sin(\beta) d\beta d\alpha d\gamma$$

$$\int_{\alpha=0}^{2\pi} \int_{\beta=0}^{\pi/2} \int_{\gamma=0}^{2\pi} f(\mathbf{g}) d\mathbf{g} = 8\pi^2$$

From Pole figures to the ODF



Pole figure: one direction fixed in K_A



Orientation: two directions fixed in K_A

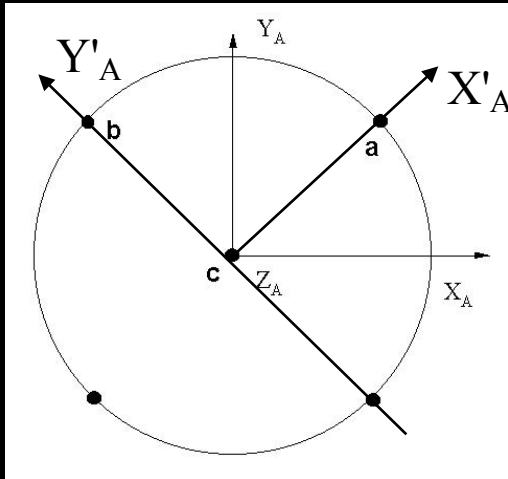
Fundamental Equation of QTA

$$P_h(\mathbf{y}) = \frac{1}{2\pi} \int_{h//y} f(g) d\tilde{\varphi}$$

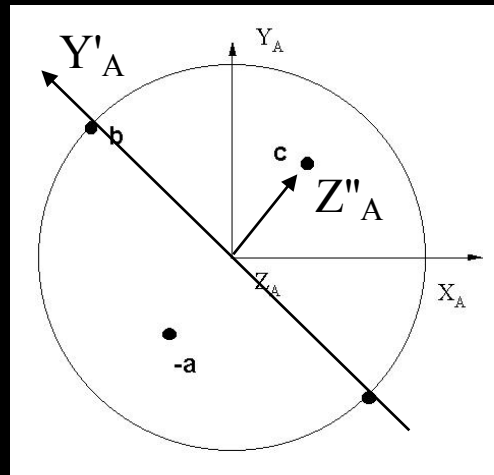
Needs several pole figures to construct $f(g)$

Pole figures from g

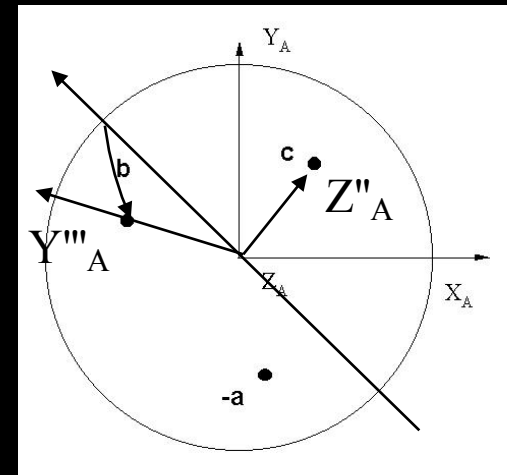
- Rotation of K_A about the axis Z_A through the angle α :
 $[K_A \mapsto K'_A]$; associated rotation $g_1 = \{\alpha, 0, 0\}$
 - Rotation of K'_A about the axis Y'_A through the angle β :
 $[K'_A \mapsto K''_A]$; associated rotation $g_2 = \{0, \beta, 0\}$
 - Rotation of K''_A about the axis Z''_A through the angle γ :
 $[K''_A \mapsto K'''_A // K_B]$; associated rotation $g_3 = \{0, 0, \gamma\}$
- finally: $g = g_1 g_2 g_3 = \{\alpha, \beta, \gamma\}$



$$g_1 = \{45, 0, 0\}$$



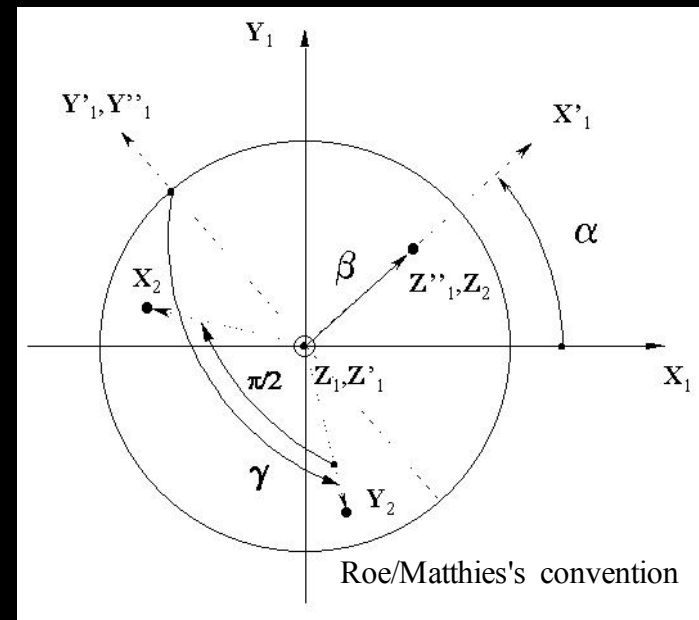
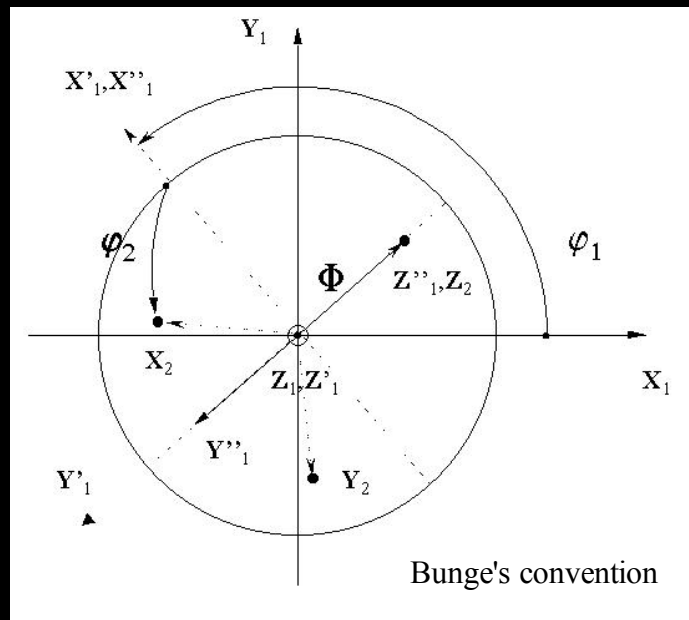
$$g_2 = \{45, 45, 0\}$$



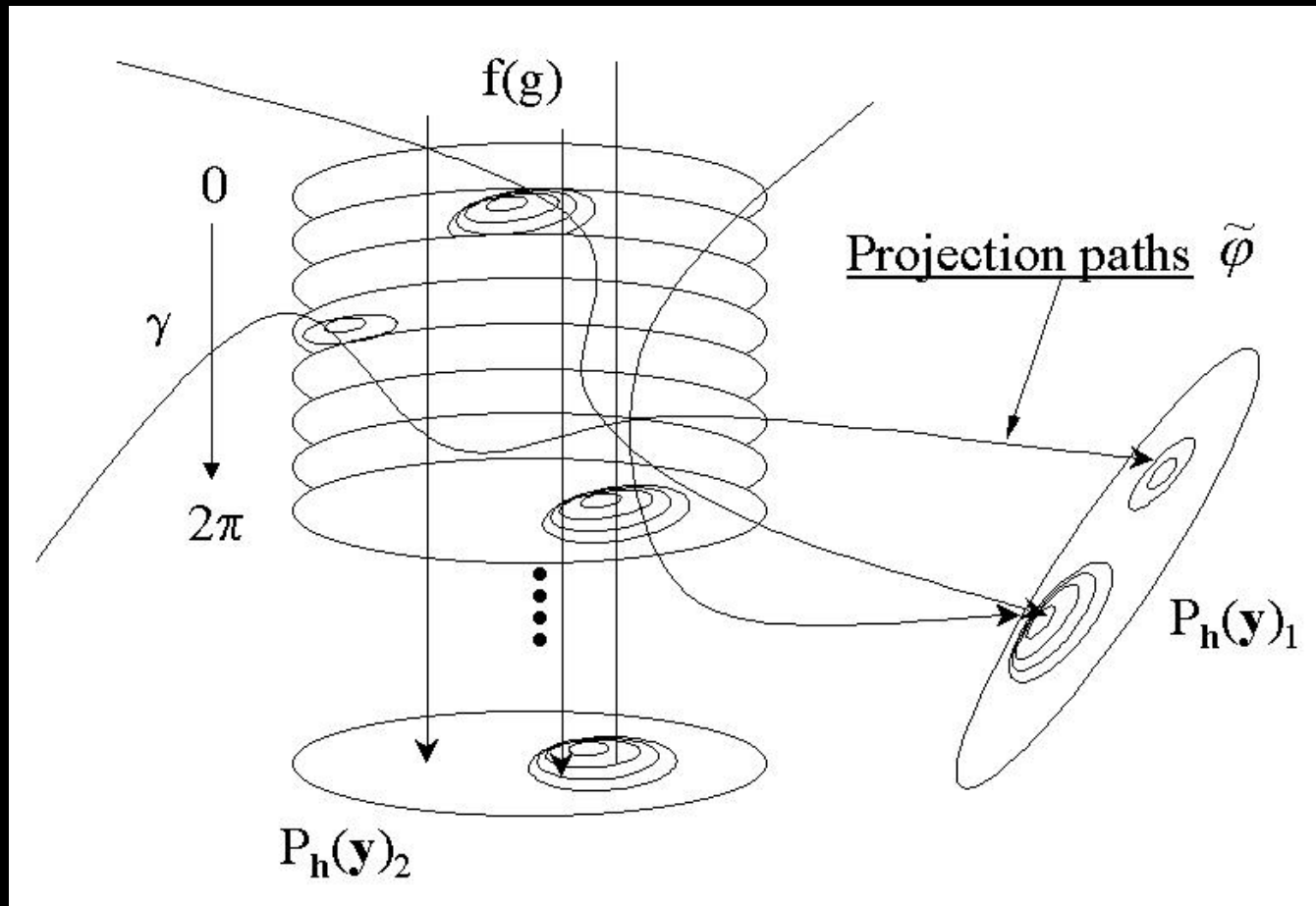
$$g_3 = \{45, 55, 45\}$$

Euler angles conventions

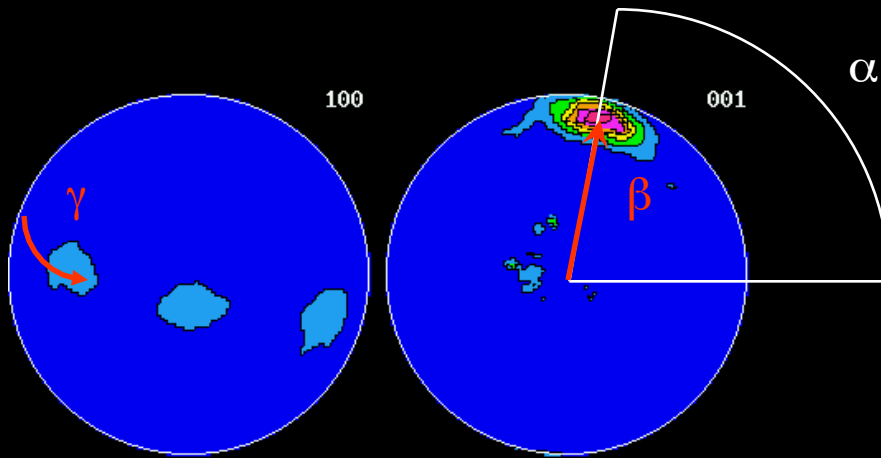
Matthies	Roe	Bunge	Canova	Kocks
α	Ψ	$\varphi_1 = \alpha + \pi/2$	$\omega = \pi/2 - \alpha$	Ψ
β	Θ	Φ	Θ	Θ
γ	Φ	$\varphi_2 = \gamma + 3\pi/2$	$\phi = 3\pi/2 - \gamma$	$\Phi = \pi - \gamma$



From $f(g)$ to the pole figures



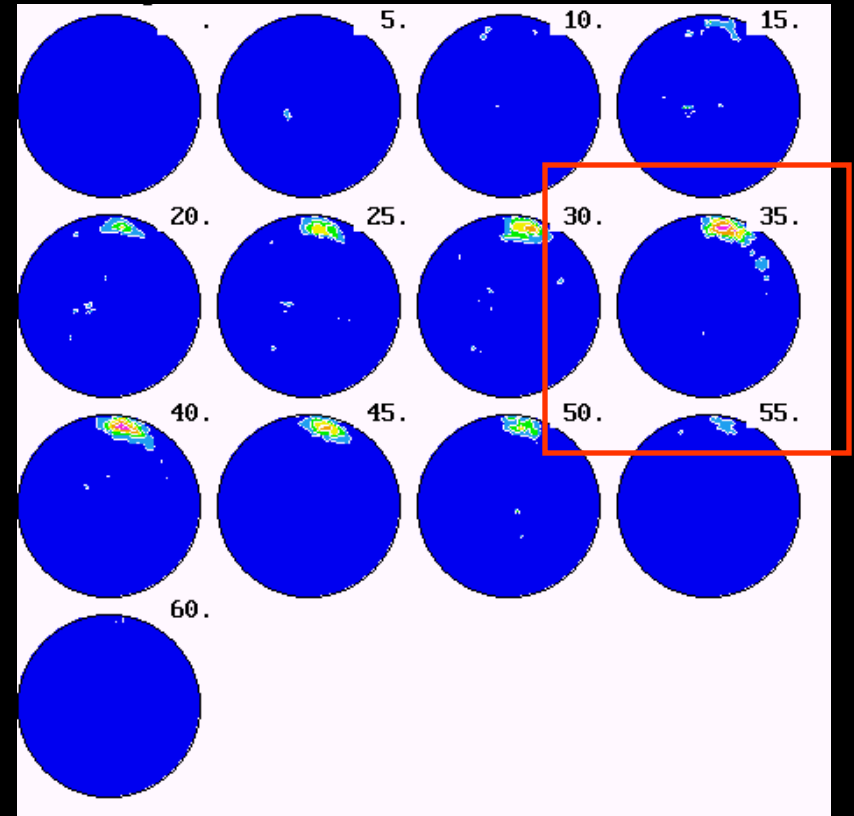
Deal with components in the ODF space



Pole figures

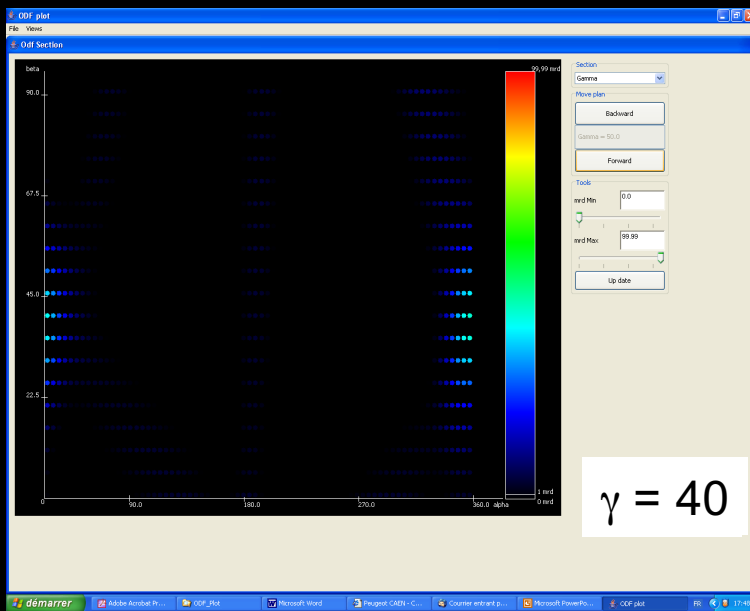
Component:
(Hexagonal system)
 $g = \{85, 80, 35\}$

ODF γ -sections

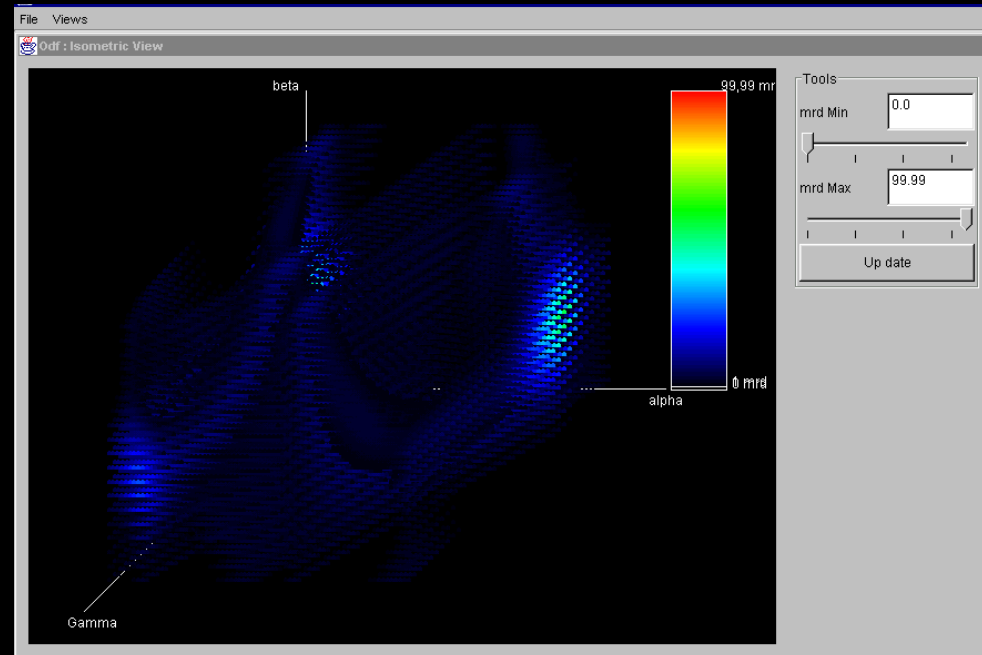


Plotting f(g)

A 3D plotting program: ODF plot



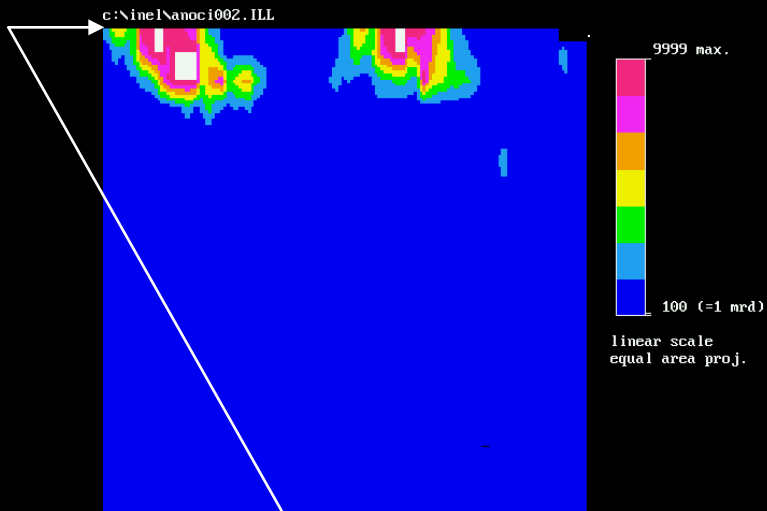
ODF sections (α , β , or γ)



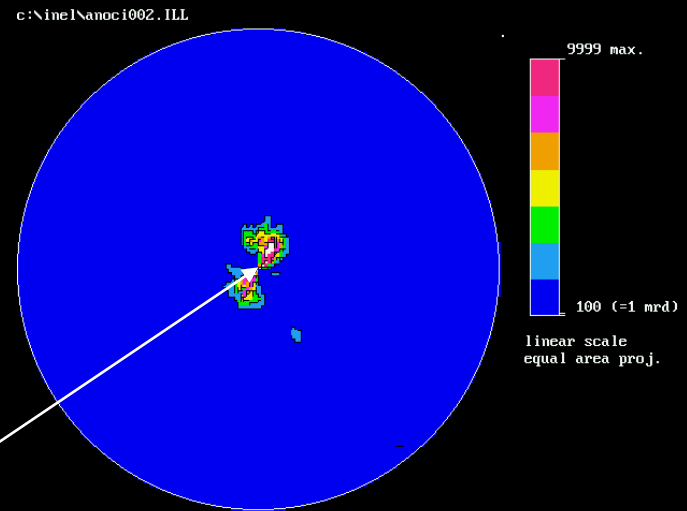
ODF 3D-isometric view

Cartesian or Polar f(g) view

Cartesian



Polar



$\beta = 0$: space deformation

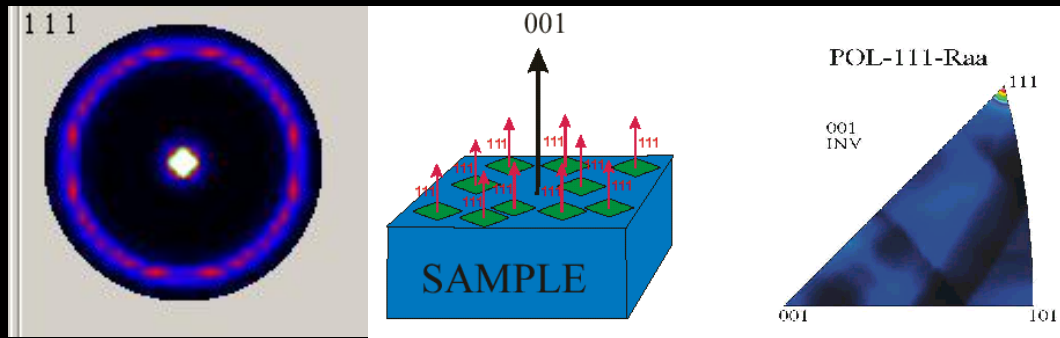
Inverse pole figures

$$P_h(\mathbf{y}) = \frac{1}{2\pi} \int_{h//y} f(g) d\tilde{\varphi}$$

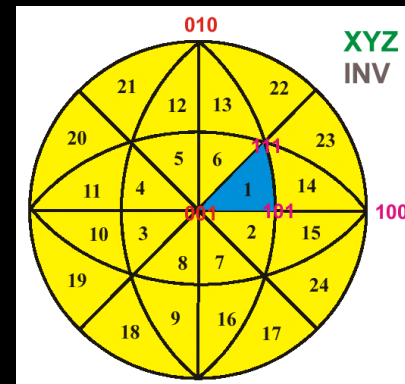
Pole figures

$$R_y(\mathbf{h}) = \frac{1}{2\pi} \int_{y//h} f(g) d\tilde{\varphi}$$

Inverse Pole figures



24 equivalent cubic sectors for the Inverse pole figure of a cubic system



ODF refinement

One has to invert:

$$P_h(\mathbf{y}) = \frac{1}{2\pi} \int_{h//\mathbf{y}} f(\mathbf{g}) d\tilde{\varphi}$$

• from Generalized Spherical Harmonics (Bunge):

$$f(\mathbf{g}) = \sum_{l=0}^{\infty} \sum_{m,n=-l}^l C_l^{mn} T_l^{mn}(\mathbf{g})$$

$$P_h(\mathbf{y}) = \sum_{l=0}^{\infty} \frac{1}{2l+1} \sum_{n=-l}^l k_l^n(\mathbf{y}) \sum_{m=-l}^l C_l^{mn} k_n^{*m}(\Theta_h \phi_h)$$

Least-squares Refinement procedure

$$\sum_h \sum_y [I_h(\mathbf{y}) - N_h P_h(\mathbf{y})]^2 d\mathbf{y}$$

But even orders are the only available parts:

$$f^e(\mathbf{g}) = \sum_{\lambda=0(2)}^{\infty} \sum_{m,n=-\lambda}^{\lambda} C_{\lambda}^{mn} T_{\lambda}^{mn}(\mathbf{g})$$

- from the WIMV iterative process (Williams-Imhof-Matthies-Vinel):

$$f^{n+1}(g) = N_n \frac{f^n(g) f^0(g)}{\left(\prod_{h=1}^I \prod_{m=1}^{M_h} P_h^n(\mathbf{y}) \right)^{\frac{1}{M_h}}}$$

and

$$f^0(g) = N_0 \left(\prod_{h=1}^I \prod_{m=1}^{M_h} P_h^{\text{exp}}(\mathbf{y}) \right)^{\frac{1}{M_h}}$$

E-WIMV (Rietveld only):

with $0 < r_n < 1$, relaxation parameter,
 M_h number of division points of the integral around k ,
 w_h reflection weight

$$f^{n+1}(g) = f^n(g) \prod_{m=1}^{M_h} \left(\frac{P_h(\mathbf{y})}{P_h^n(\mathbf{y})} \right)^{r_n \frac{w_h}{M_h}}$$

- Entropy maximisation (Schaeben) and exponential harmonics (van Houtte):

$$f^{n+1}(g) = f^n(g) \prod_{m=1}^{M_h} \left(\frac{P_h(\mathbf{y})}{P_h^n(\mathbf{y})} \right)^{\frac{r_n}{M_h}}$$

$$f_s(g) = e^{h(g)} \geq 0$$

$$C_{s\lambda}^{mn} = (2\lambda + 1) \int e^{h(g)} T_\lambda^{mn}(g) dg$$

- arbitrarily defined cells (ADC, Pawlik):

Very similar to E-WIMV, with integrals along path tubes

- Vector method (Ruer, Baro, Vadon):

I linear equations for J unknown quantities:

$$\mathbf{P}_i(\mathbf{h}) = [\sigma_{ij}(\mathbf{h})] f_j$$

- Component method (Helming):

$$f(g) = F + \sum_c I^c f^c(g)$$

Gaussian component:

$$f(g, g^c) = f(\tilde{g}) = \frac{2\sqrt{\pi}}{\xi \left\{ 1 - \exp\left(-\left(\frac{\xi}{2}\right)^2\right)\right\}} \exp\left(-\left(\frac{\tilde{g}}{\xi}\right)^2\right)$$

$$S = \frac{\ln 2}{1 - \cos\left(\frac{\xi}{2}\right)}$$

$$N(S) = \frac{1}{I_0(S) - I_1(S)}$$

Evaluation of the OD coverage

Say 20 measured ($5^\circ \times 5^\circ$) complete pole figures:

= $20 \times 1368 = 27360$ experimental points

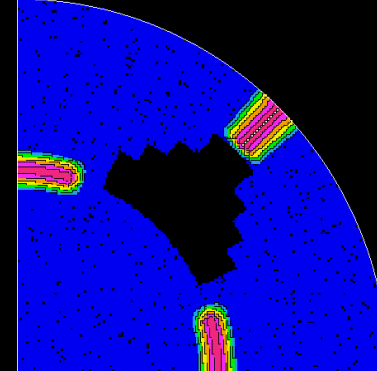
ODF ($5^\circ \times 5^\circ \times 5^\circ$, triclinic): 98496 points to refine

{100} pole figure, measured up to $\chi = 45^\circ$:

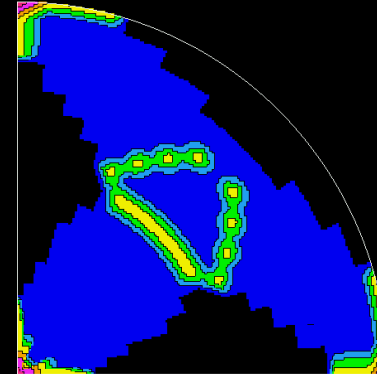
{100} + {110}, measured up to $\chi = 45^\circ$:

{100} + {110} + {111}, up to $\chi = 45^\circ$:

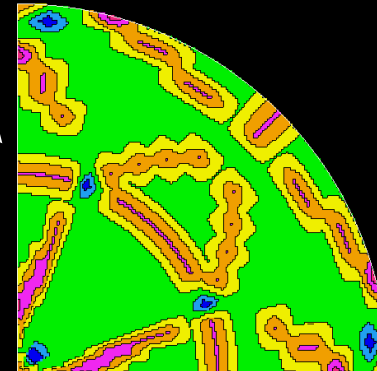
Evaluation of ODF coverage



Evaluation of ODF coverage

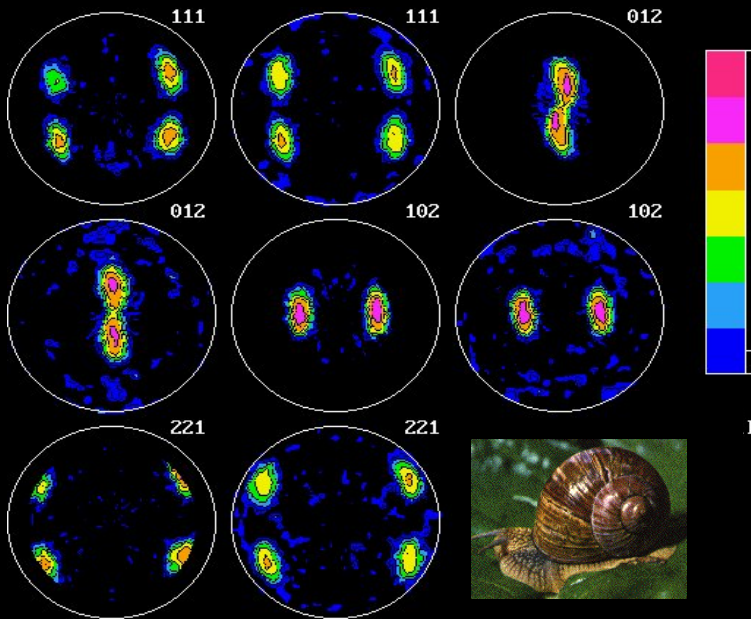


Evaluation of ODF coverage

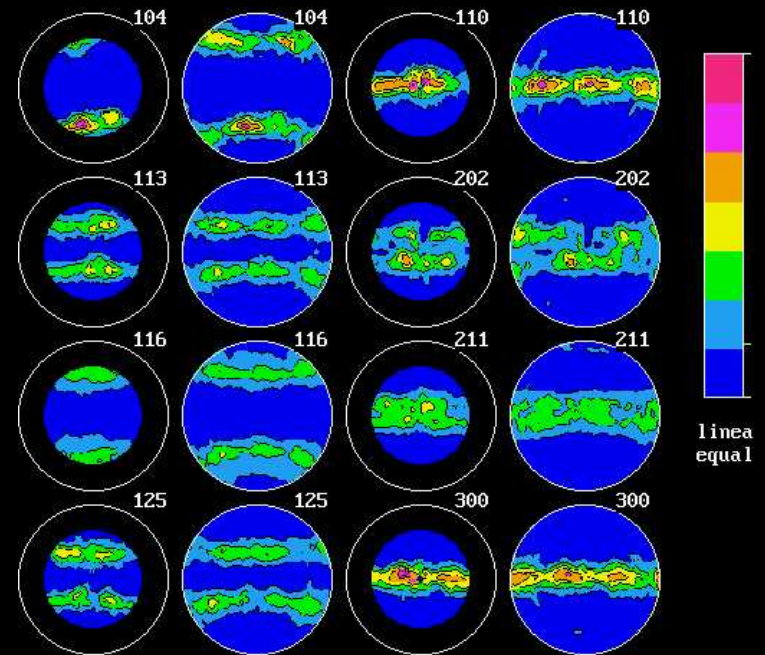


Estimators of Refinement Quality

Visual assessment



Helix pomatia (Burgundy land snail:
Outer com. crossed lamellar layer)



Bathymodiolus thermophilus (deep
ocean mussel: Outer Prismatic layer)

RP Factors:

Individual pole figures:

$$RP_x(h_i) = \frac{\sum_{j=1}^J |\tilde{P}_{h_i}^o(y_j) - \tilde{P}_{h_i}^c(y_j)|}{\sum_{j=1}^J \tilde{P}_{h_i}^o(y_j)} \theta(x, \tilde{P}_{h_i}^o(y_j))$$

$$\theta(x, t) = \begin{cases} 1 & \text{for } t > x \\ 0 & \text{for } t \leq x \end{cases}$$

$x = \varepsilon, 1, 10 \dots$

Averaged on all pole figures:

$$\overline{RP}_x = \frac{1}{I} \sum_{i=1}^I RP_x(h_i)$$

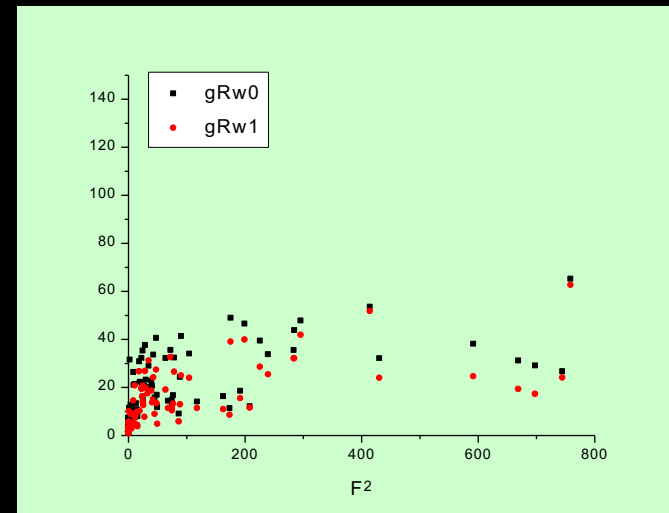
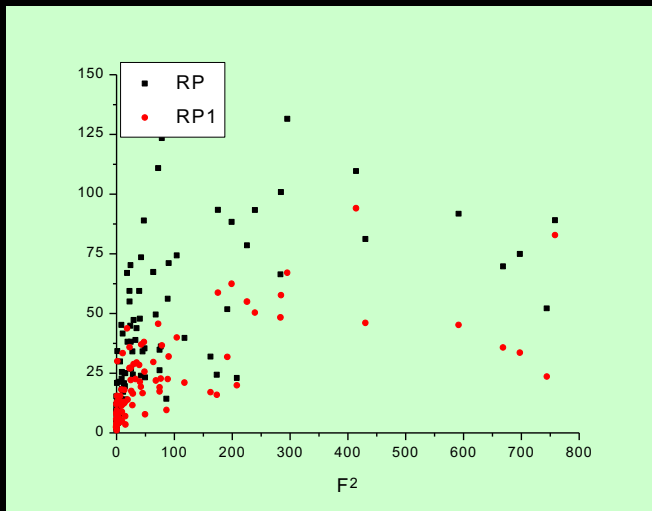
Bragg R-Factors:

$$RB_x(h_i) = \frac{\sum_{j=1}^J [\tilde{P}_{h_i}^o(y_j) - \tilde{P}_{h_i}^c(y_j)]^2}{\sum_{j=1}^J \tilde{P}_{h_i}^o{}^2(y_j)} \theta(x, \tilde{P}_{h_i}^o(y_j))$$

Weighted Rw-Factors:

$$w_{ij} = \frac{1}{\sqrt{I_{h_i}^o(y_j)}}$$

$$Rw_x(h_i) = \frac{\sum_{j=1}^J [w_{ij}^o I_{h_i}^o(y_j) - w_{ij}^c I_{h_i}^c(y_j)]^2}{\sum_{j=1}^J w_{ij}^o{}^2 I_{h_i}^o{}^2(y_j)} \theta(x, \tilde{P}_{h_i}^o(y_j))$$



Texture strength estimators

ODF Texture Index:

$$F^2 \in [1, \infty[\quad \begin{array}{l} > 1 \text{ m.r.d}^2 \\ = 1: \text{ powder} \\ = \infty: \text{ single crystal} \end{array}$$

$$F^2 (\text{m.r.d.}^2) = \frac{1}{8\pi^2} \sum_i f^2(g_i) \Delta g_i$$

Discrete OD

$$F^2 = 1 + \sum_{\lambda=2}^L \left[\frac{1}{2\lambda+1} \right] \sum_{m=-\lambda}^{\lambda} \sum_{n=-\lambda}^{\lambda} |C_{\lambda}^{mn}|^2$$

Continuous ODF

Pole figures Texture Index:

$$J_h^2 = \frac{1}{4\pi} \sum_i [P_h(\mathbf{y}_i)]^2 \Delta \mathbf{y}_i$$

Texture Entropy:

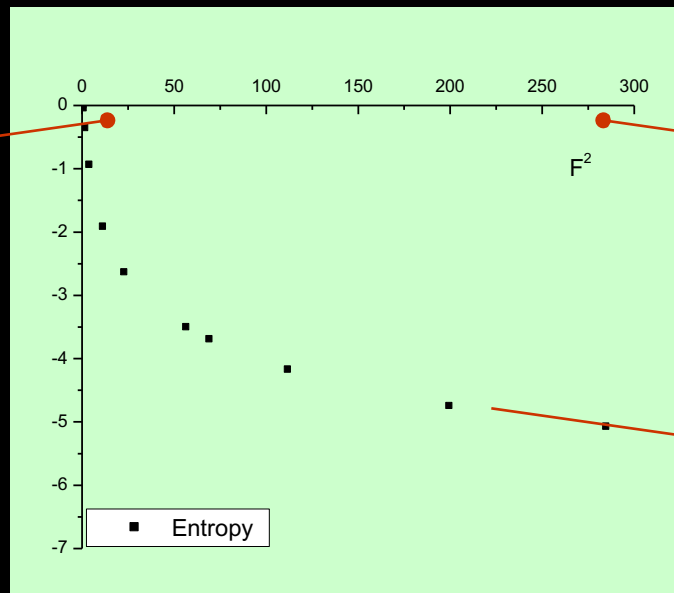
$$S \in [0, -\infty[\quad \leq 0$$

= 0: powder
= $-\infty$: single crystal

$$S = \frac{-1}{8\pi^2} \sum_i f(g_i) \ln[f(g_i)] \Delta g_i$$

S - F²:

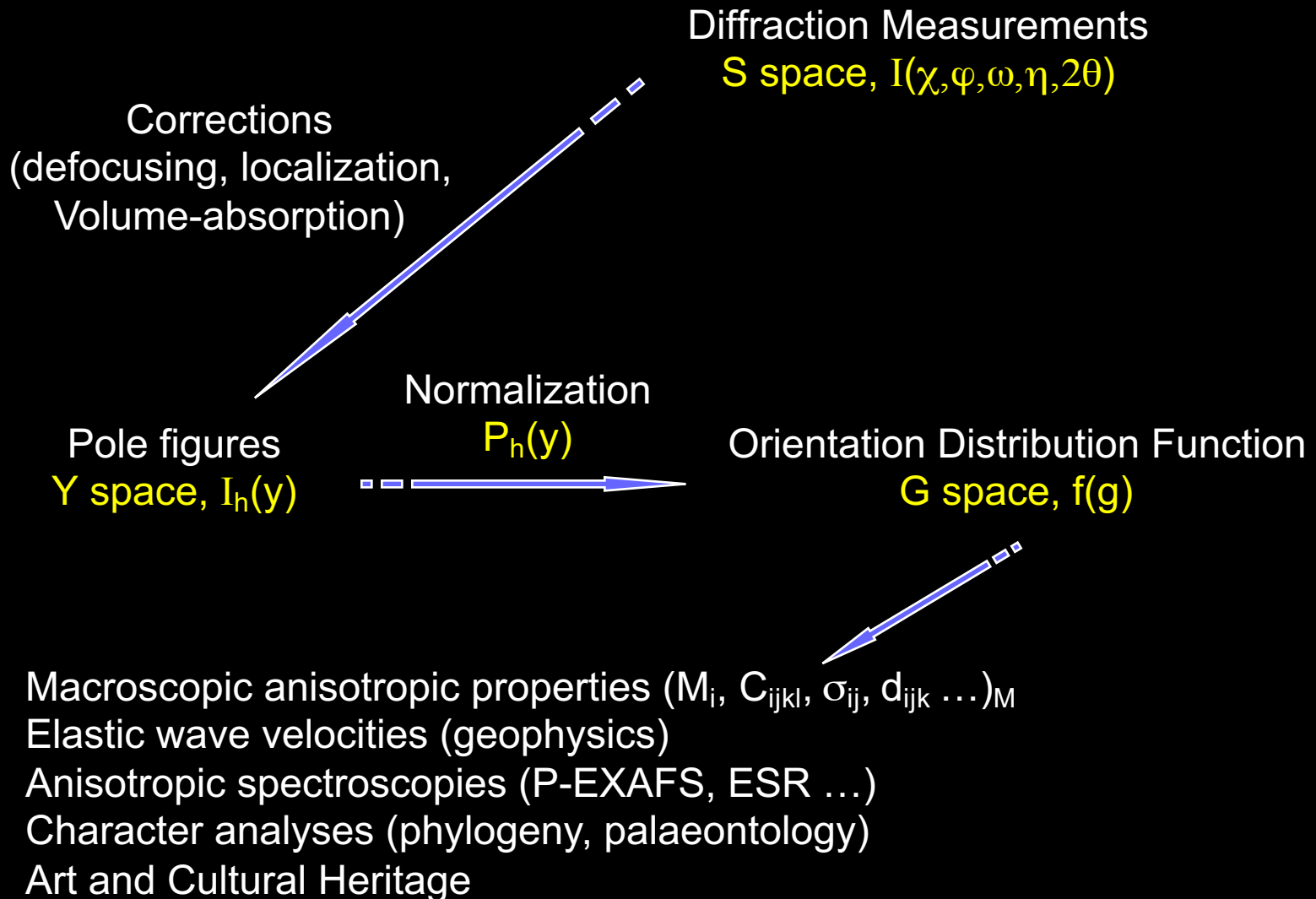
Fon +
smooth
texture component(s)



Fon +
Dirac-like
texture component

Lower bound:
Fon = 0

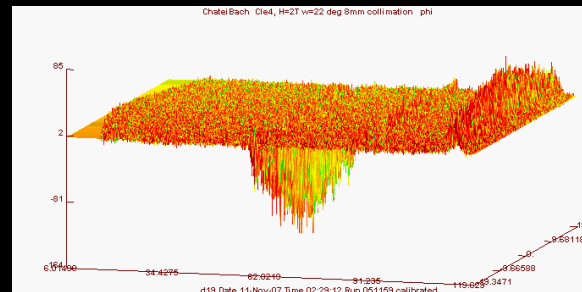
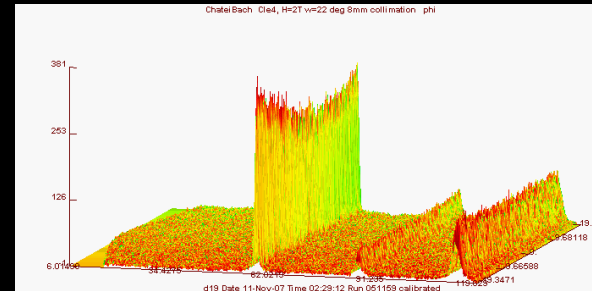
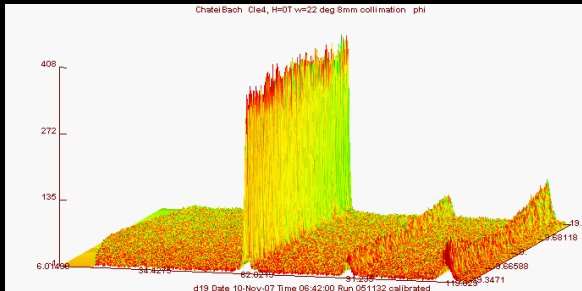
Crystallographic texture



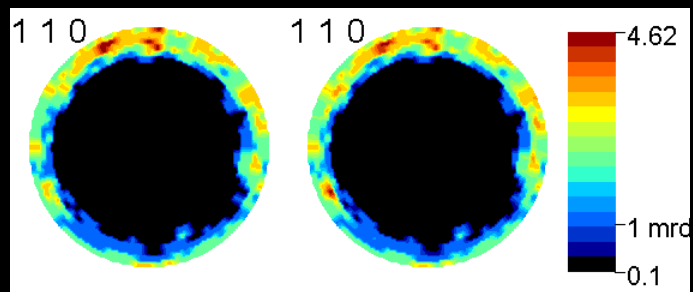
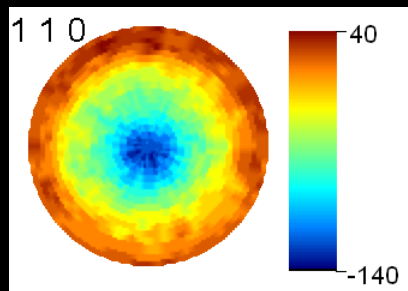
Magnetic QTA

$$I_{\vec{h}}(\vec{y}, 0) = I_{\vec{h}}^n(\vec{y}, 0) + I_{\vec{h}}^m(\vec{y}, 0)$$

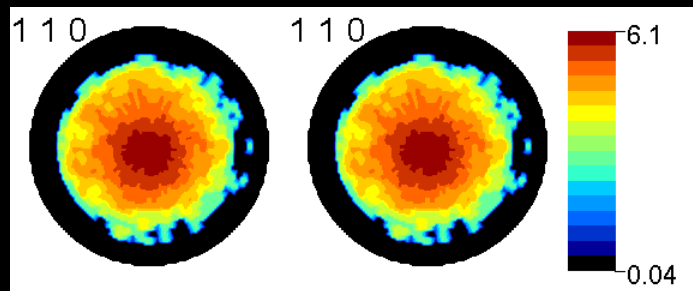
$$I_{\vec{h}}(\vec{y}, \vec{B}) = I_{\vec{h}}^n(\vec{y}, 0) + I_{\vec{h}}^m(\vec{y}, \vec{B})$$



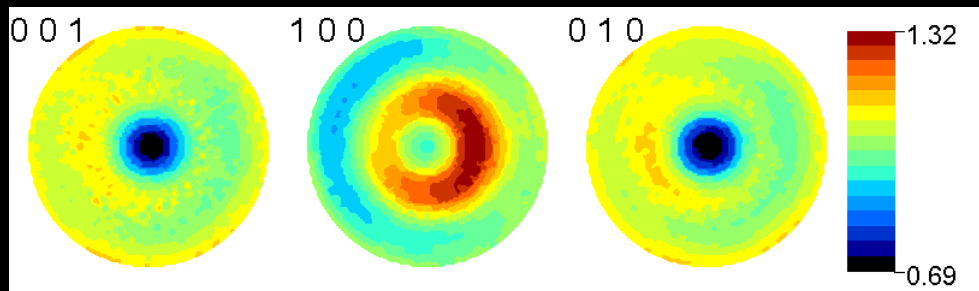
$$\Delta I_{\vec{h}}^m(\vec{y}, \vec{B}) = I_{\vec{h}}(\vec{y}, \vec{B}) - I_{\vec{h}}(\vec{y}, 0)$$



$$\Delta I_{\vec{h}}^{+p}(\vec{y}, \vec{B})$$



$$\Delta I_{\vec{h}}^{-p}(\vec{y}, \vec{B})$$

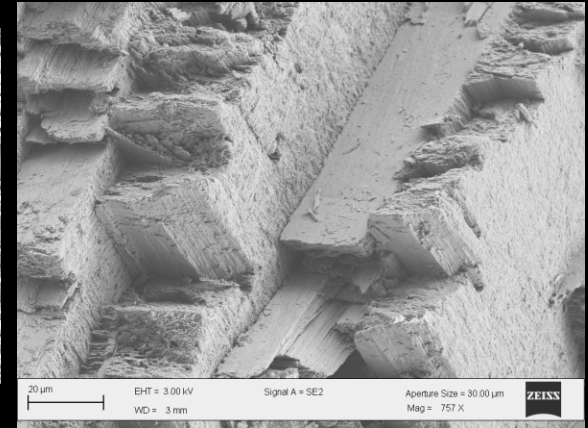
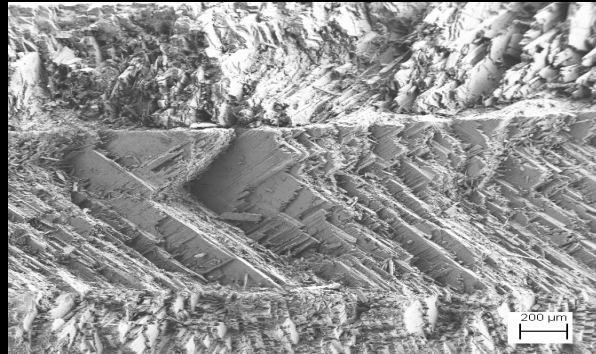


**** True iteration step #120 ****

ODF min max:	0.64	2.26
Texture Index (F^2)	1.0294	
Entropy	-0.0144	
Average RP	0.2427	
Average RP1	0.3041	

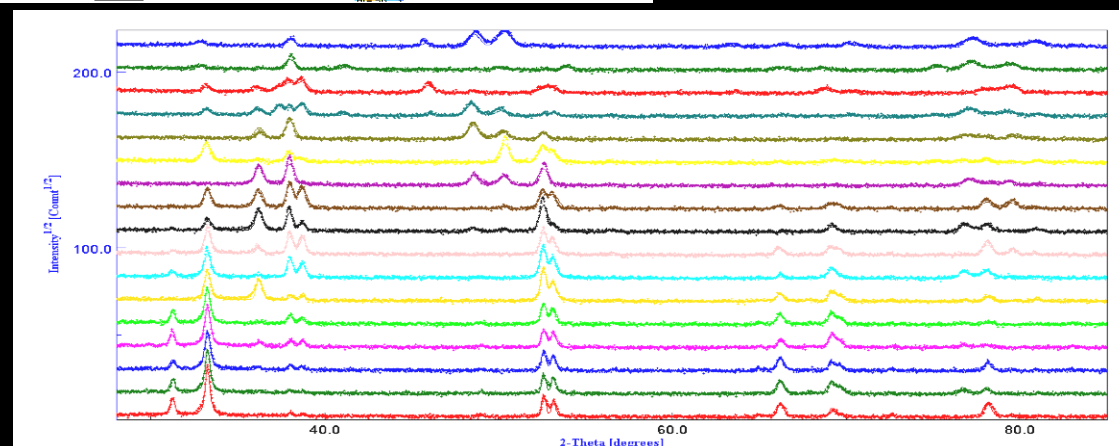
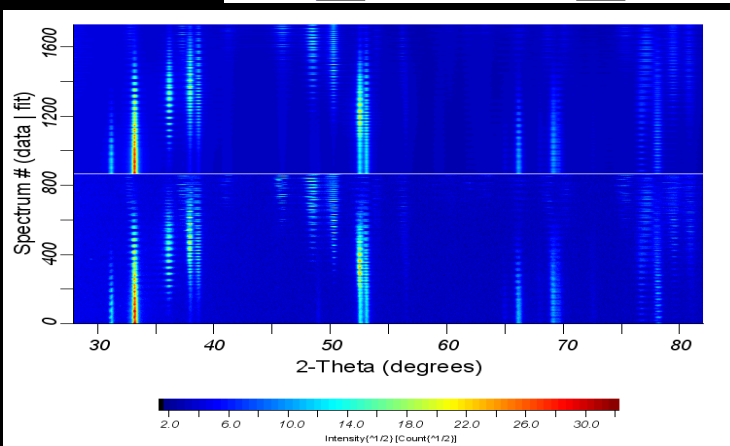
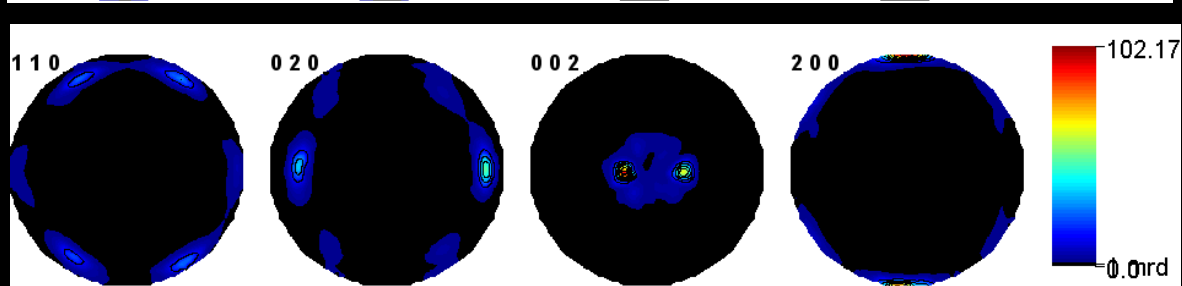
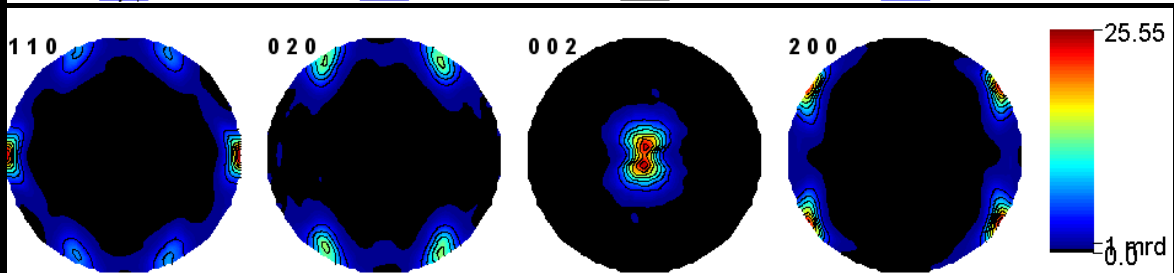
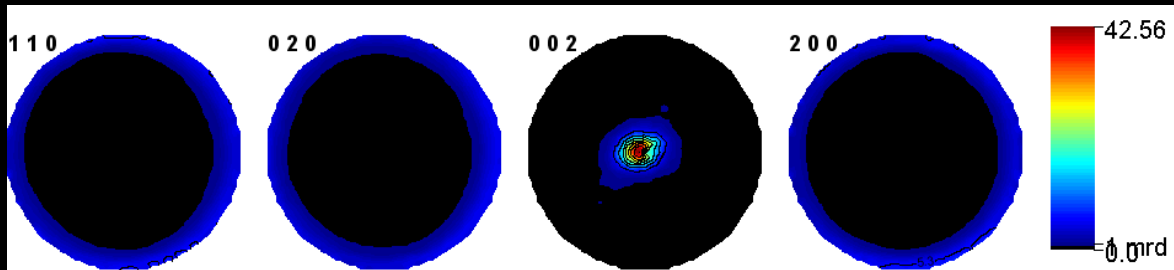
Why needing QTA

- Correct for QTA effects in XRD: structure analysis
- QTA and structure **correlations**: yes, but **$f(g)$ and $|F_h|^2$ are different !**



Charonia lampas lampas

OD maximum (m.r.d.)		299	196	2816
OD minimum (m.r.d.)		0	0	0
Texture index (m.r.d. ²)		42.6	47	721
Texture reliability factors	R _w (%)	14.3	11.2	32.5
	R _B (%)	15.6	12.7	47.8
Rietveld reliability factors	GoF (%)	1.72	1.72	3.05
	R _w (%)	29.2	28.0	57.3
	R _B (%)	22.9	21.7	47.2
	R _{exp} (%)	22.2	21.3	32.8

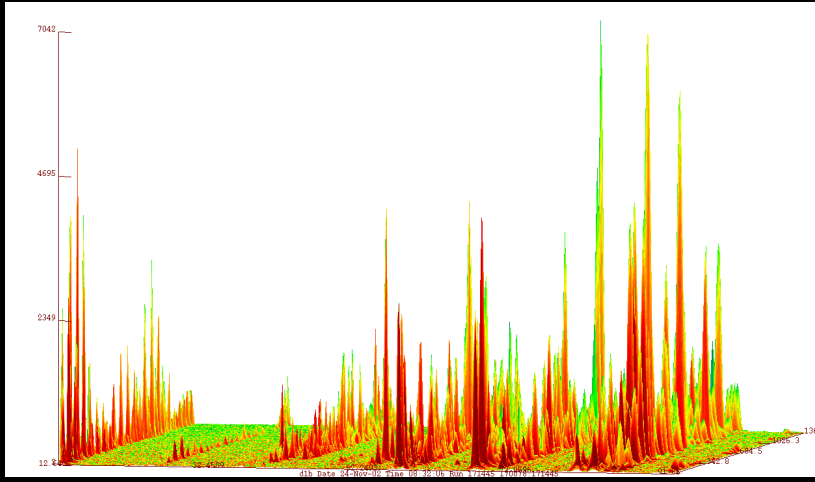


		Geological reference	<i>Charonia lampas</i> OCL	<i>Charonia lampas</i> RCL	<i>Charonia lampas</i> ICCL	Strombus decorus
a (Å)		4.9623(3)	4.98563(7)	4.97538(4)	4.9813(1)	4.9694(3)
b (Å)		7.968(1)	8.0103(1)	7.98848(8)	7.9679(1)	7.9591(4)
c (Å)		5.7439(3)	5.74626(3)	5.74961(2)	5.76261(5)	5.7528(1)
Ca	y	0.41500	0.41418(5)	0.414071(4)	0.41276(9)	0.4135(7)
	z	0.75970	0.75939(3)	0.76057(2)	0.75818(8)	0.7601(8)
C	y	0.76220	0.7628(2)	0.76341(2)	0.7356(4)	0.7607(4)
	z	-0.08620	-0.0920(1)	-0.08702(9)	-0.0833(2)	-0.0851(7)
O1	y	0.92250	0.9115(2)	0.9238(1)	0.8957(3)	0.9228(4)
	z	-0.09620	-0.09205(8)	-0.09456(6)	-0.1018(2)	-0.0905(9)
O2	x	0.47360	0.4768(1)	0.4754(1)	0.4864(3)	0.4763(6)
	y	0.68100	0.6826(1)	0.68332(9)	0.6834(2)	0.6833(3)
	z	-0.08620	-0.08368(6)	-0.08473(5)	-0.0926(1)	-0.0863(7)
ΔZ_{C-O1} (Å)		0.05744	0.00029	0.04335	0.1066	0.031

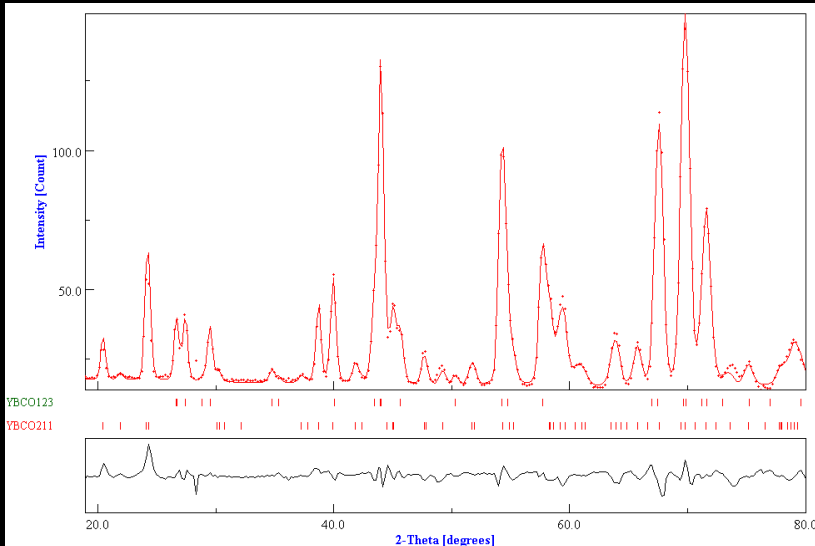
Calcite: $\Delta Z = 0$

Biogenic intercrystalline effect

- Correct for QTA effects in XRD: QPA
QTA and QPA **correlations**: yes, but
 $f(g)$ and S_{Φ} are different !

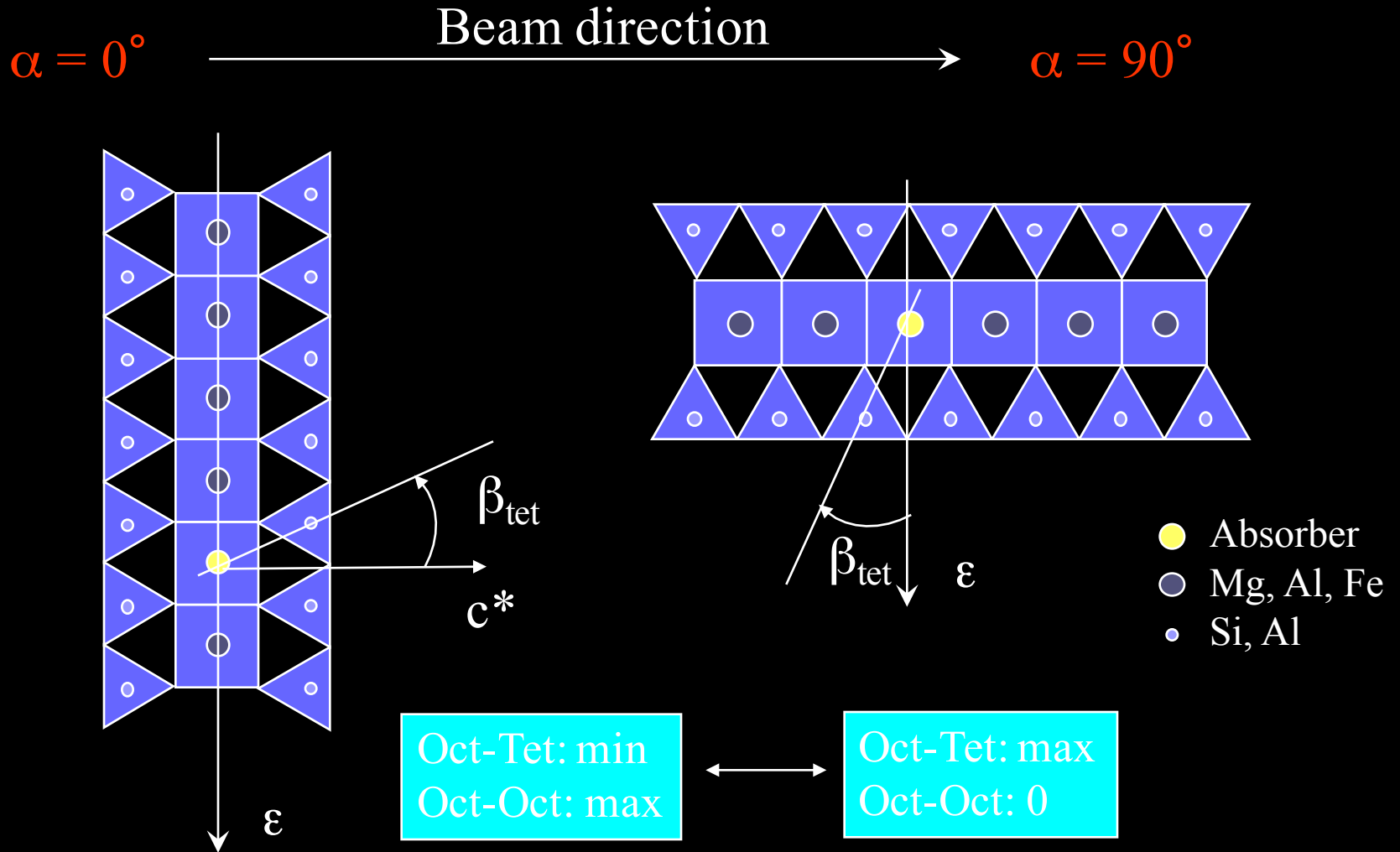


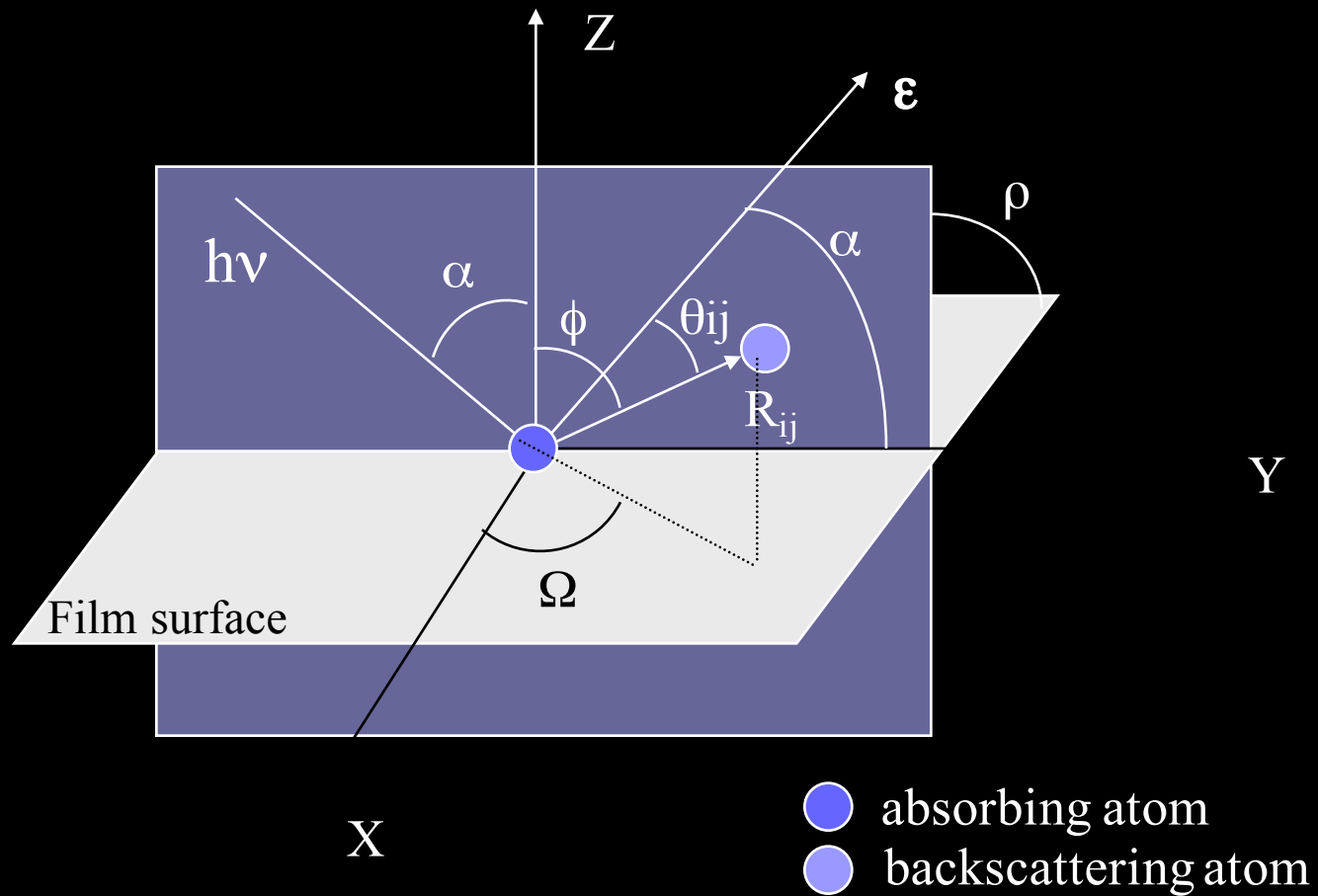
$f(g)$ is on the individuals



S_{Φ} is on the sum

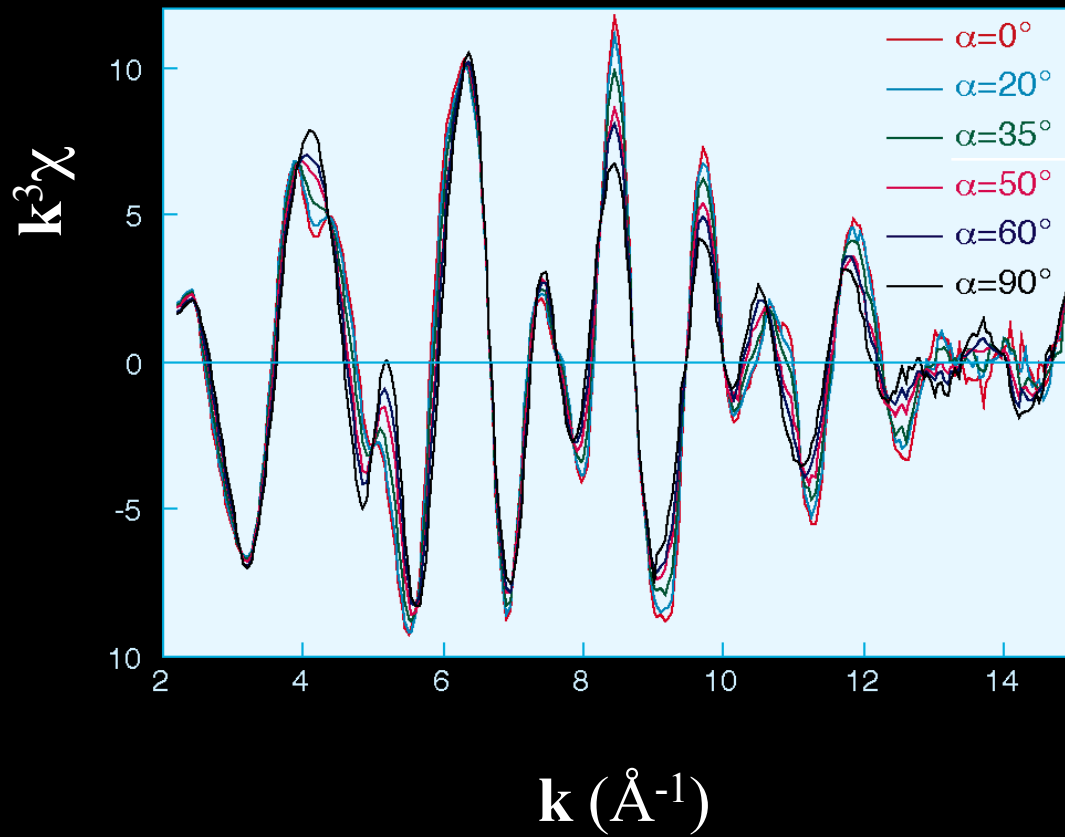
- Correct for QTA effects in spectroscopies: P-EXAFS on clays





$$\langle \cos^2 \theta_{ij} \rangle = \frac{1}{2\pi} \int_0^{2\pi} \cos^2 \theta_{ij} d\Omega = \cos^2 \phi \sin^2 \alpha + \frac{\cos^2 \alpha \sin^2 \phi}{2}$$

Fe K-edge

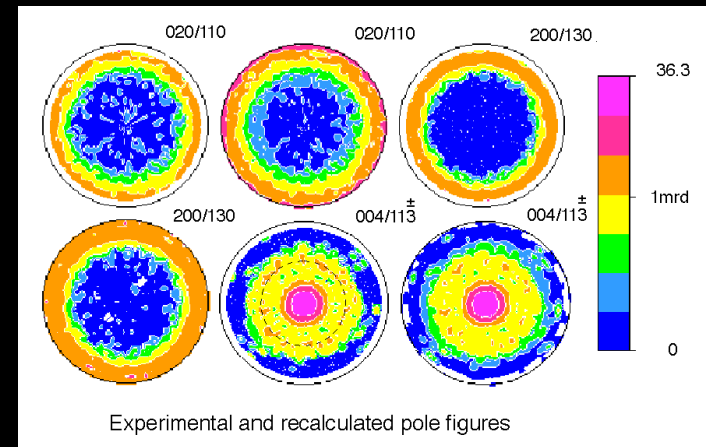


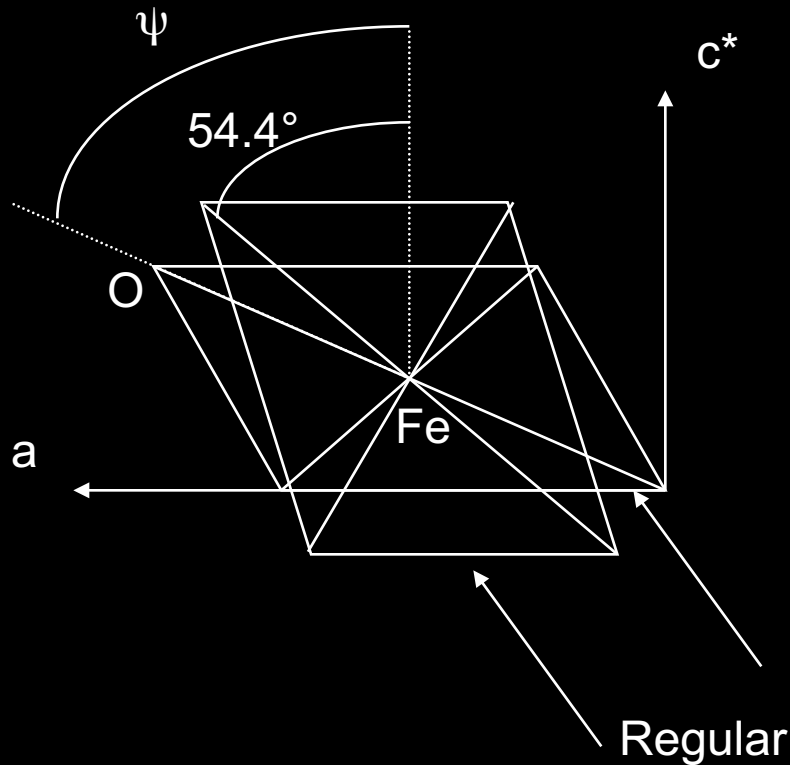
High quality range up to 14-15 Å⁻¹

Powder spectra

Strong α dependence
= strong texture

$$N_{obs} = 3N_{real} \left[\cos^2 \phi \sin^2 \alpha + \frac{\cos^2 \alpha \sin^2 \phi}{2} \right]$$



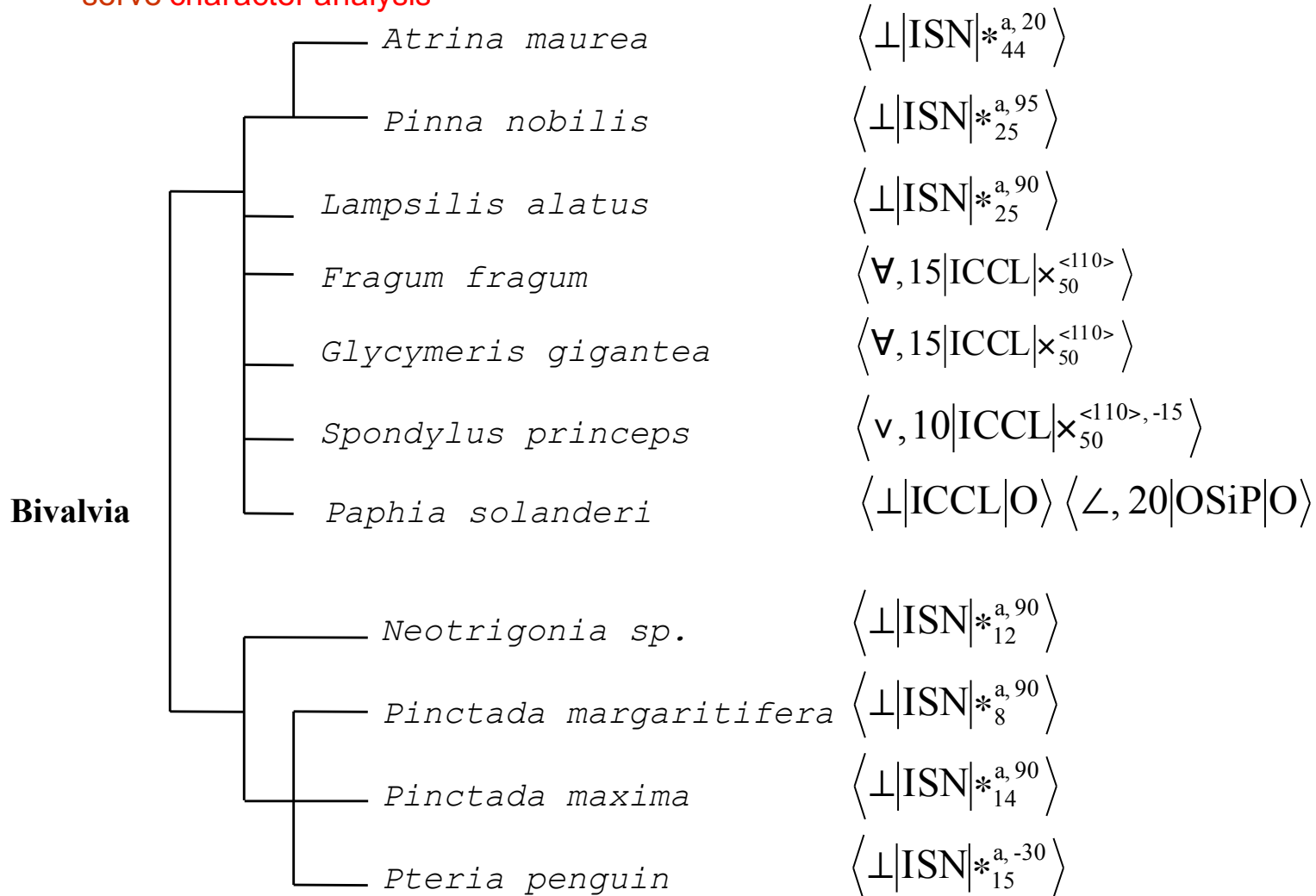


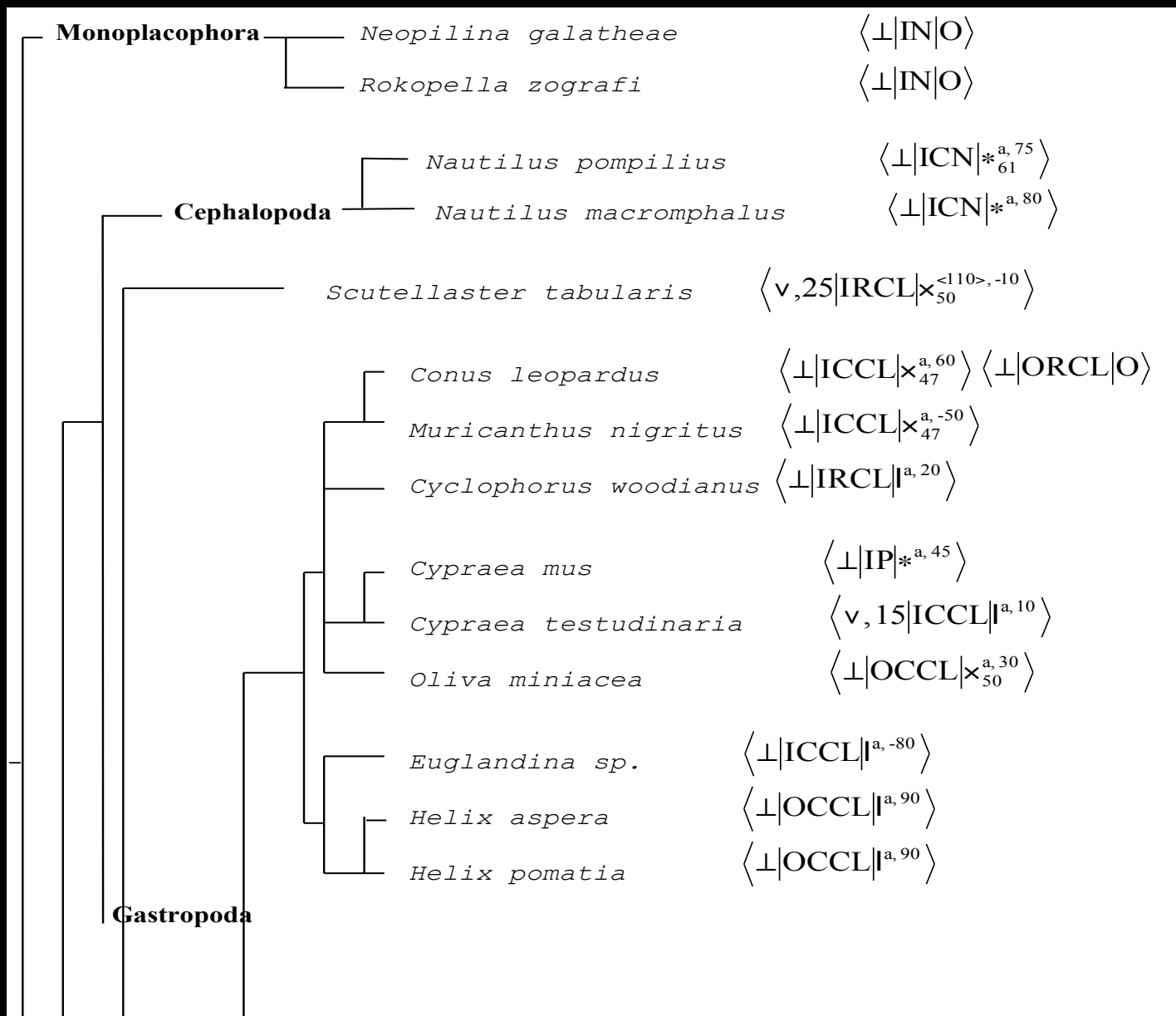
For ideally textured films:

$$\frac{I_\alpha}{I_0} = \frac{1 + \frac{1}{2}(3 \sin^2 \alpha - 1)(3 \cos^2 \psi - 1)}{1 - \frac{1}{2}(3 \cos^2 \psi - 1)}$$

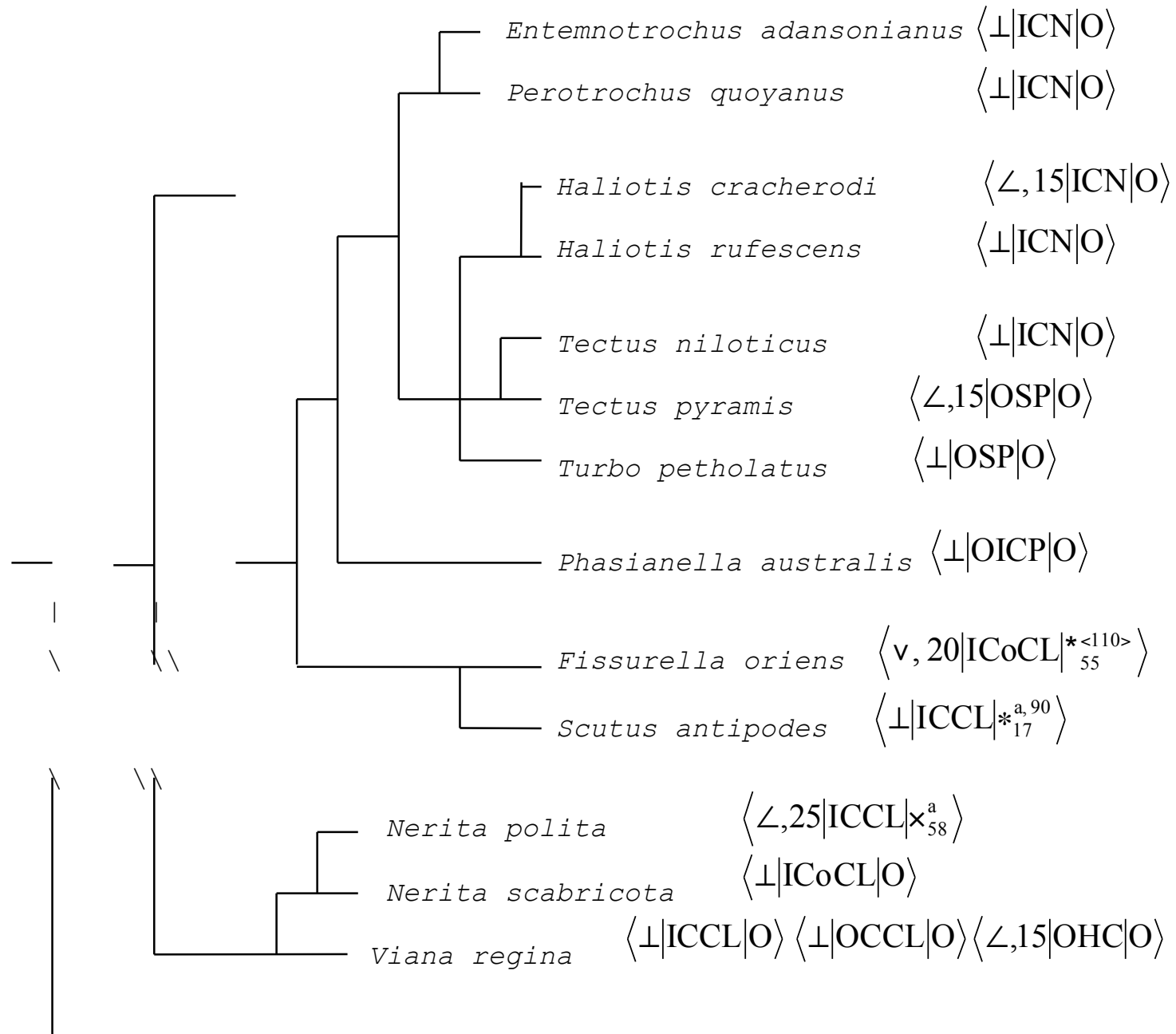
- Mollusc shells and fossils: phylogeny

Closely related species, close textural characters, but significant variations: **textural parameters can serve character analysis**

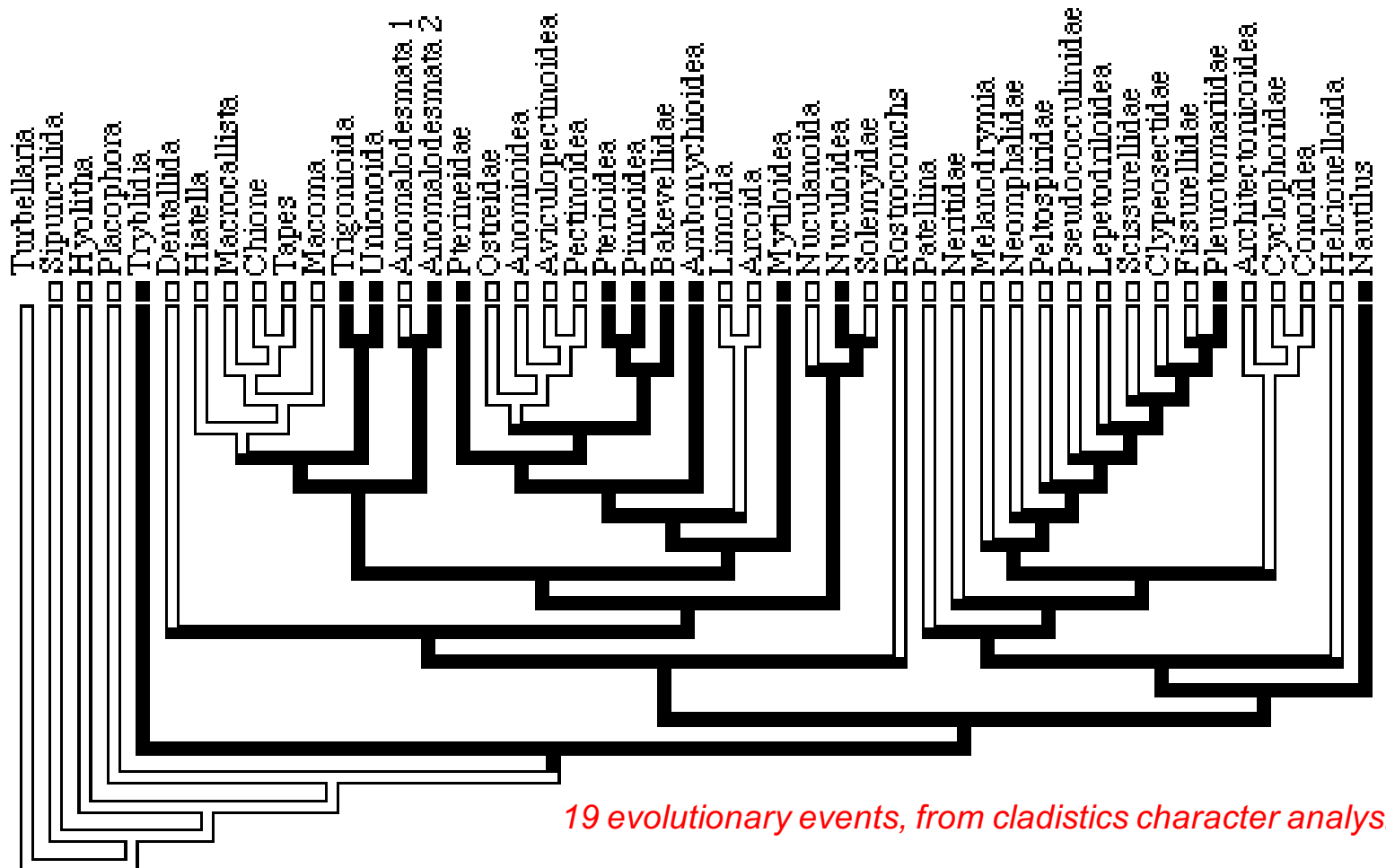




Gastropoda

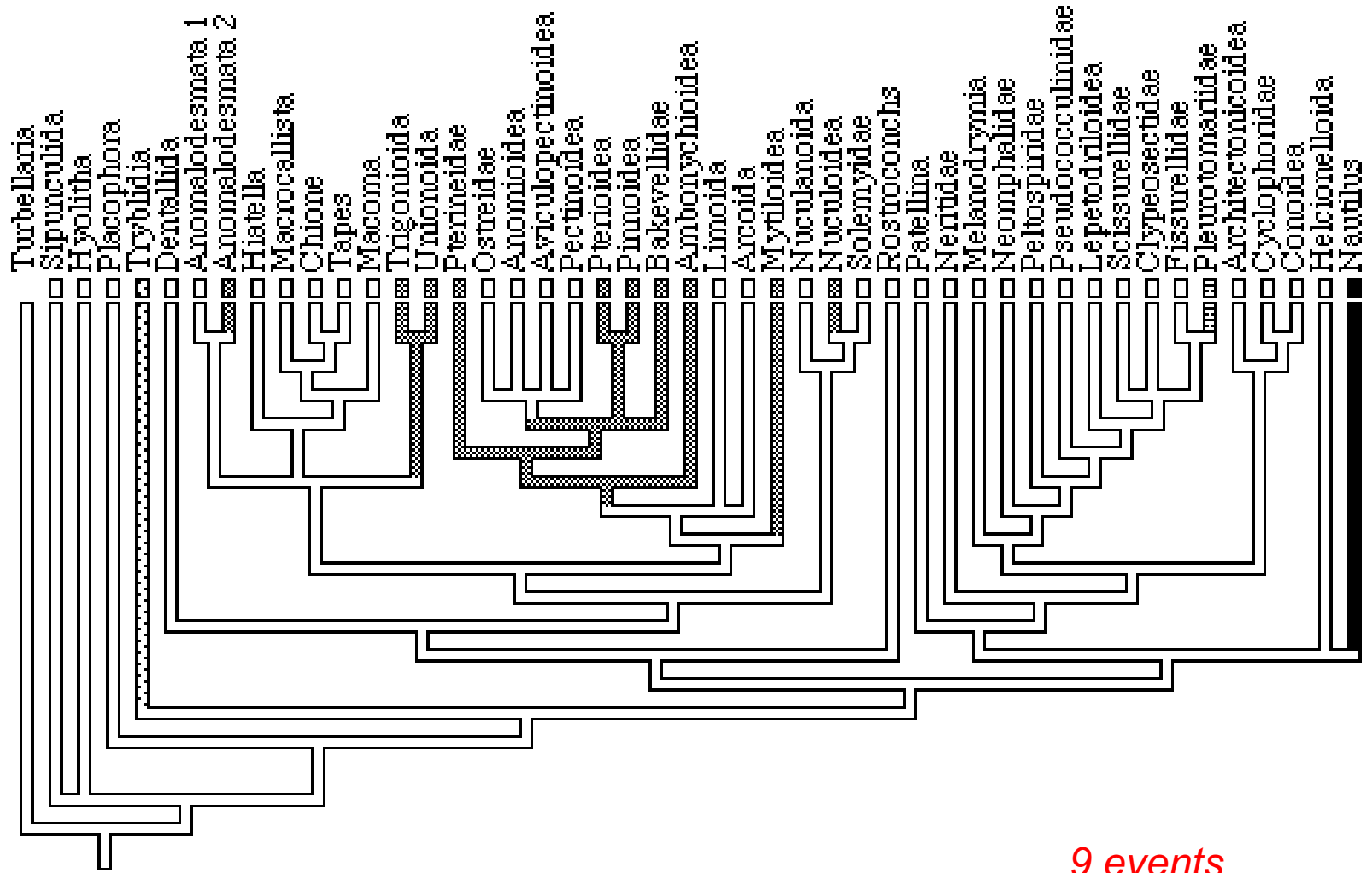


Phylogenetic interest: nacre = ancestral (Carter & Clarck, 1985)



19 evolutionary events, from cladistics character analysis

nacre not ancestral

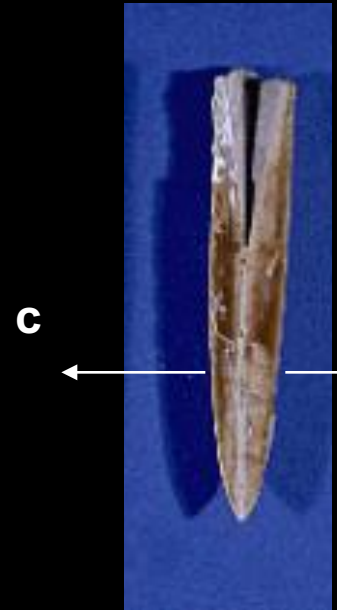
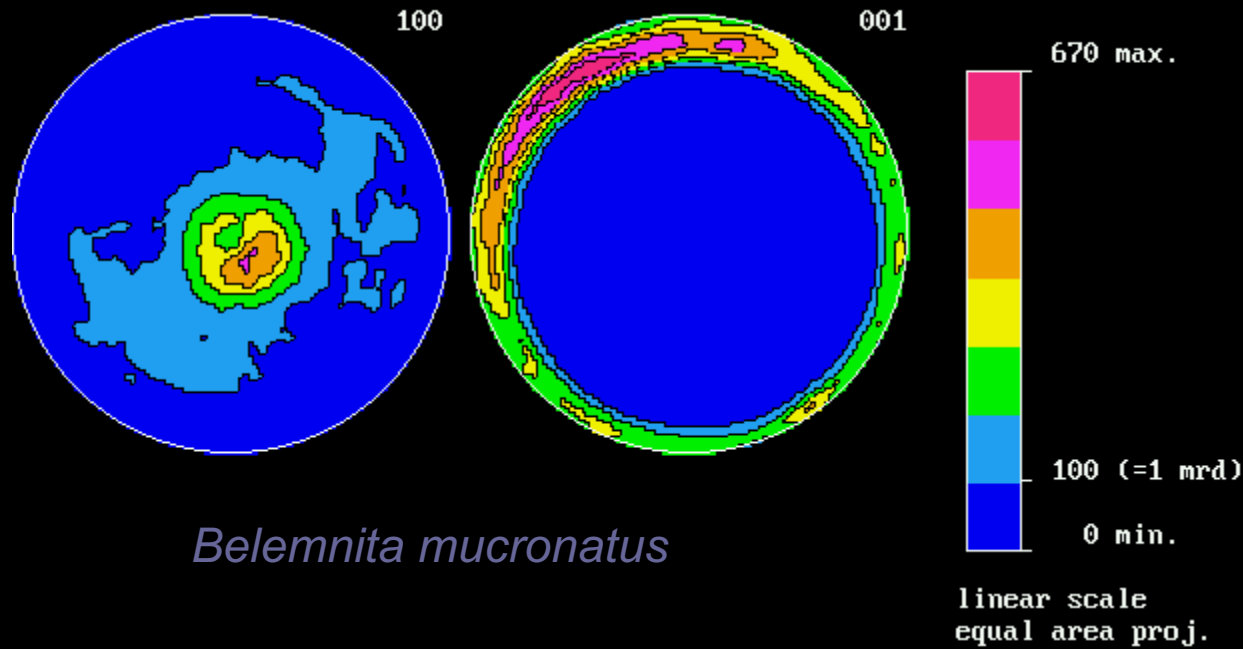


Calcitic fossils: trichites

	Layer type	ODF Max (mrd)	ODF min (mrd)	RP0 (%)	RP1 (%)	c-axis	a-axis	{001} Max (mrd)	F ² (mrd ²)	- S
<i>Pinna nobilis</i>	OP	303	0	50	29	// N	random	68	29	2.3
<i>Pteria penguin</i>	OP	84	0	29	15	// N	random	31	13	1.9
<i>Amussium parpiraceum</i>	OP	330	0	53	33	// G	<110> // M	20	31	2.6
<i>Bathymodiolus thermophilus</i>	OP	63	0	25	18	// G	// M	27	13	1.9
<i>Mytilus edulis</i>	OP	207	0	41	25	75° from N	<110> // M	23	21	2.2
<i>Trichites</i>	P	390	0	52	28	15° from N	random	56	41	2.2
<i>Crassostrea gigas</i>	IF	908	0	45	31	35° from N	// M	>100	329	5.1

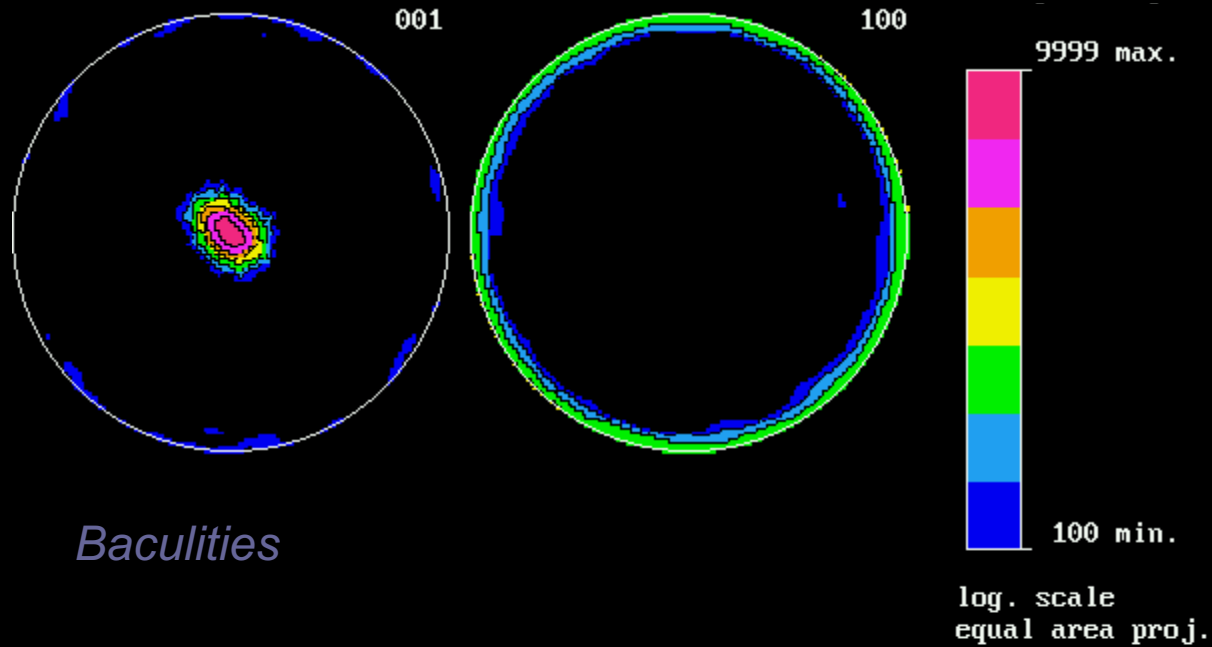
No DNA is available on fossils like Trichites, but Trichite's textural parameters are close to the ones of *pinnoids* or *pteriods*: interesting for the **classification of extinct species**

Calcitic fossils: *Belemnites*



c-axes perp. to the shell: as in other cephalopods: nacre ancestral ?

Aragonitic fossils: *Baculities* sp.



c-axes perp. to the shell: as in other cephalopods,
strong **c**-calcite to **c**-aragonite fossils interaction

- Predict macroscopic anisotropic properties: **Elastic**

Arithmetic average

$$\langle \mathcal{T} \rangle = \int_{\mathbf{g}} \mathcal{T}(\mathbf{g}) f(\mathbf{g}) d\mathbf{g}$$

$$\langle (\mathcal{T})^{-1} \rangle \neq \langle \mathcal{T} \rangle^{-1}$$

Voigt average
Homogeneous strain

$$\mathbf{C}_{ijkl}^M = \langle \mathbf{C}_{ijkl} \rangle$$

Upper bound

Reuss average
Homogeneous stress

$$\mathbf{S}_{ijkl}^M = \langle \mathbf{S}_{ijkl} \rangle$$

Lower bound

Geometric average

$$[\mathbf{b}] = \prod_{k=1}^N b_k^{w_k} = \exp(\langle \ln \mathbf{b} \rangle)$$

scalar

$$\langle \ln \mathbf{b} \rangle = \sum_{k=1}^N \ln b_k w_k$$

$$[T]_{ij} = \exp(\langle \ln T \rangle_{ij})$$

tensor

$$[\lambda_I] = 1 / [1/\lambda_I] = [\lambda_I^{-1}]^{-1}$$

Eigenvalues of T_{ij}

$$\langle (C_{ijkl})^{-1} \rangle = \langle C_{ijkl} \rangle^{-1}$$

- Predict macroscopic anisotropic properties: **Electric polarisation**

$$\langle \mathbf{P}_h \rangle = \frac{\iint_y \mathbf{P}_h P_h(\mathbf{y}) d\mathbf{y}}{\iint_y P_h(\mathbf{y}) d\mathbf{y}}$$

- Predict macroscopic anisotropic properties: **BAW**

Propagation equation

$$\rho \frac{\partial^2 u^i}{\partial t^2} = [C^{i\ell mn}] \frac{\partial^2 u_n}{\partial x^m \partial x^\ell}$$

**Propagation
direction**

V_P

V_{S1}

V_{S2}

[100]

$$\sqrt{\frac{c^M_{11}}{\rho}}$$

$$\sqrt{\frac{c^M_{44}}{\rho}}$$

$$\sqrt{\frac{c^M_{44}}{\rho}}$$

[110]

$$\sqrt{\frac{c^M_{11} + 2c^M_{44} + c^M_{12}}{2\rho}}$$

$$\sqrt{\frac{c^M_{11} - c^M_{12}}{2\rho}}$$

$$\sqrt{\frac{c^M_{44}}{\rho}}$$

[111]

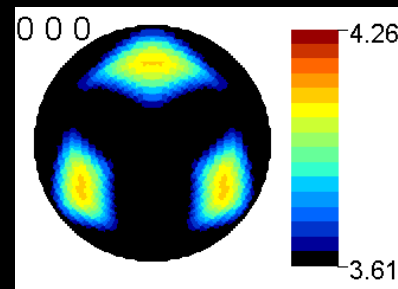
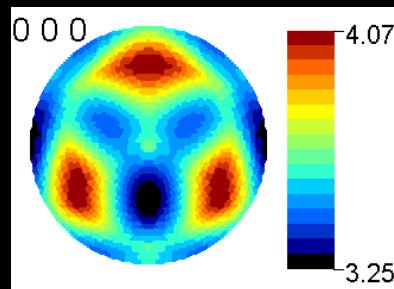
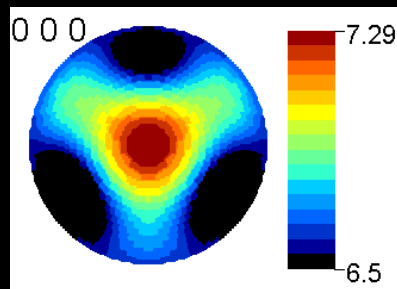
$$\sqrt{\frac{c^M_{11} + 4c^M_{44} + 2c^M_{12}}{3\rho}}$$

$$\sqrt{\frac{c^M_{11} + c^M_{44} - c^M_{12}}{3\rho}}$$

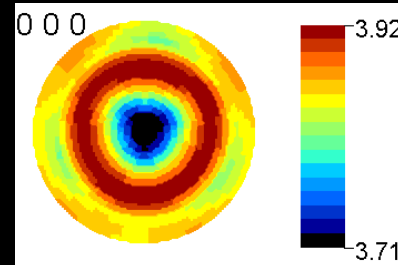
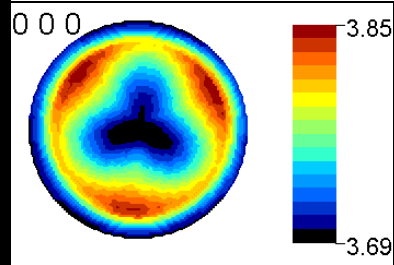
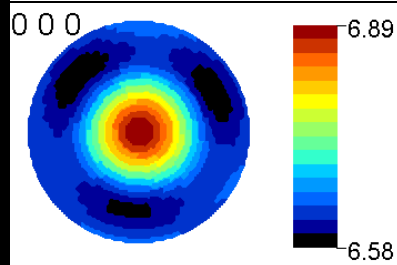
$$\sqrt{\frac{c^M_{11} + c^M_{44} - c^M_{12}}{3\rho}}$$

Cubic crystal system

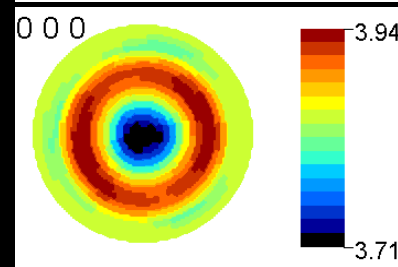
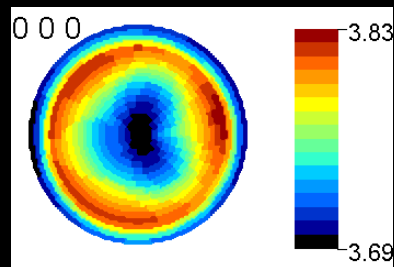
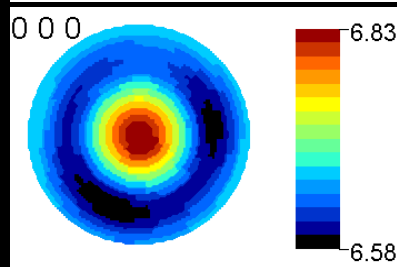
	c_{11} or c_{11}^M	c_{12} or c_{12}^M	c_{13} or c_{13}^M	c_{14} or c_{14}^M	c_{33} or c_{33}^M	c_{44} or c_{44}^M
Single crystal	201	54.52	71.43	8.4	246.5	60.55
LiNbO ₃ /Si	206.4	68.5	67.6	0.48	216.5	64
LiNbO ₃ /Al ₂ O ₃	204	65.7	69.7	1.1	219.9	63.2



Single crystal



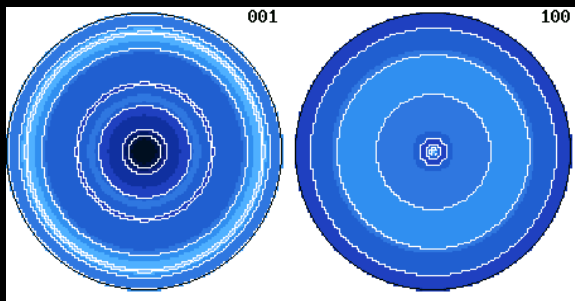
LiNbO₃/Si



LiNbO₃/Al₂O₃

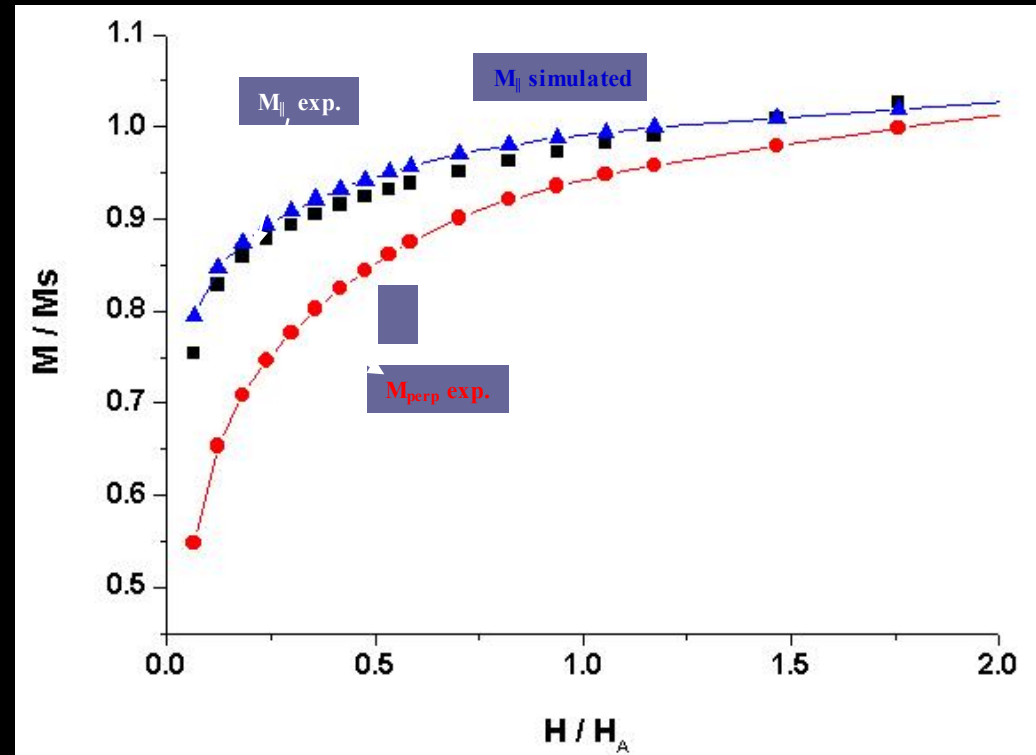
- Predict macroscopic anisotropic properties: **Magnetisation**

$$\frac{M_{\perp}}{M_S} = 2\pi \int_0^{\frac{\pi}{2}} (1 - \rho_0) PV(\theta_g) \sin\theta_g \cos(\theta_g - \theta) d\theta_g + \rho_0 M_{\text{random}}$$



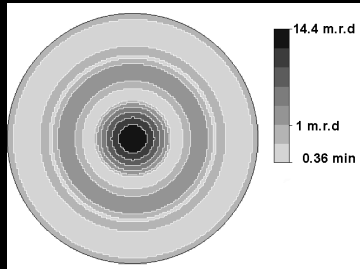
max {001}: 3.9 mrd
min: 0.5 mrd

ErMn₃Fe₉C:
ODF + micros. → macros.

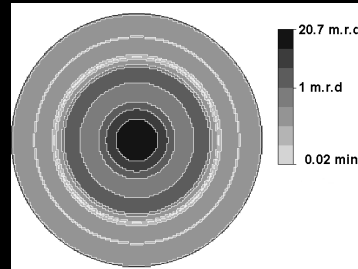


- Correlate macroscopic anisotropic properties: **Thermoelectric PF**

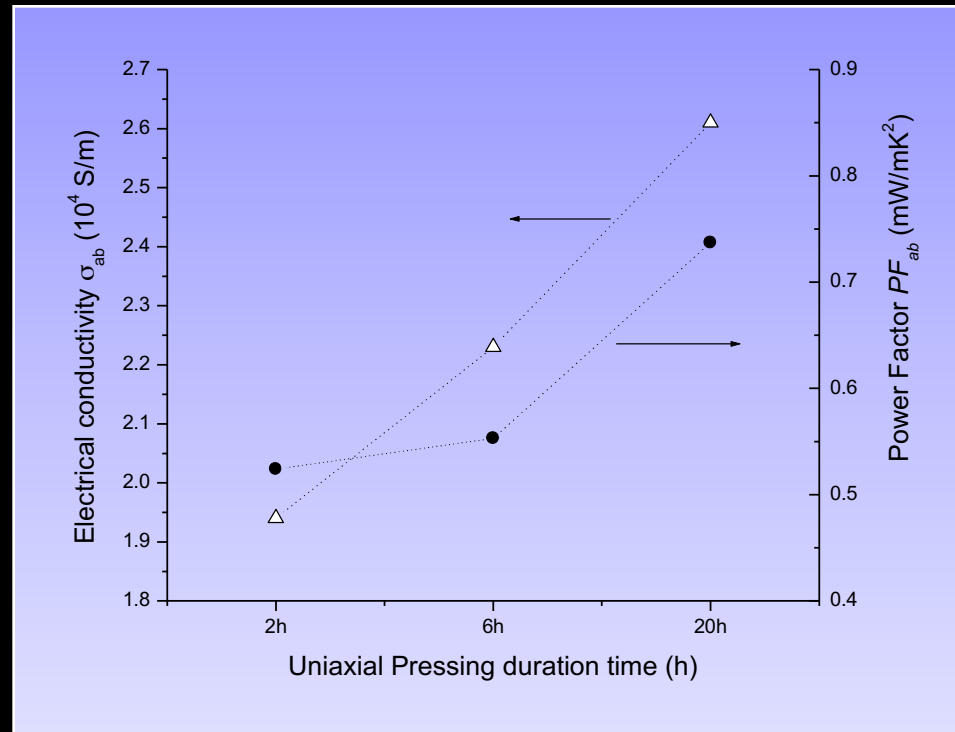
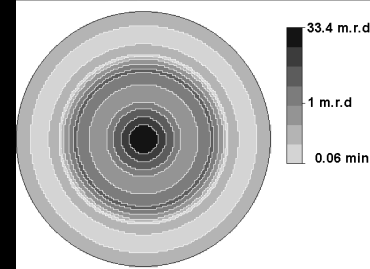
9.8 MPa for 2 h



19.6 MPa for 6 h

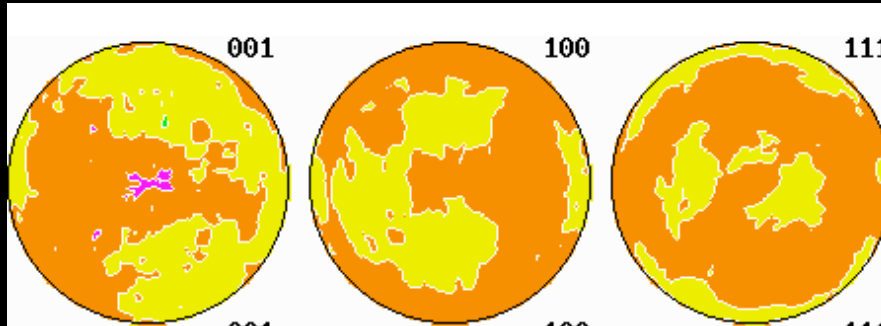


19.6 MPa for 20 h

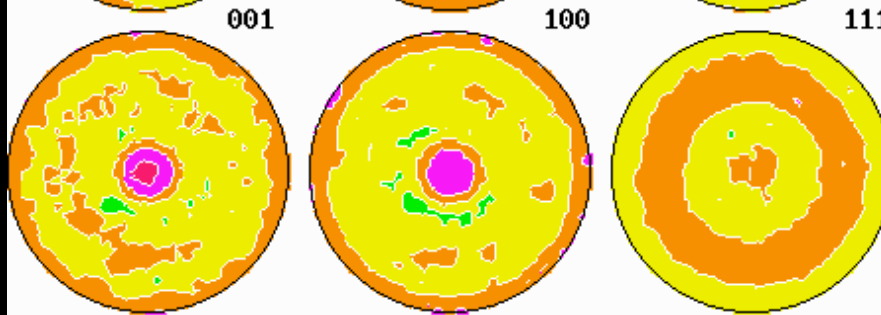


- Correlate macroscopic anisotropic properties: **Pyroelectric coefficient**

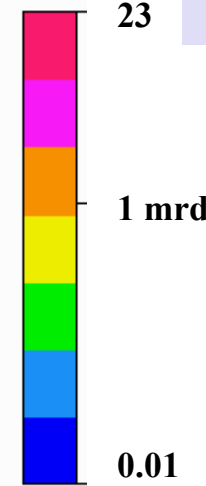
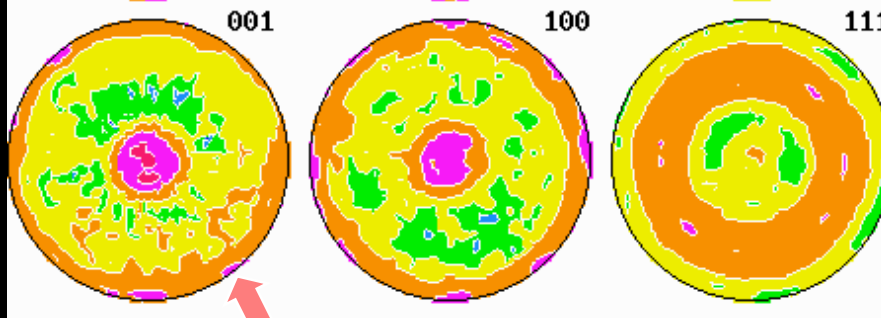
PCT on
Pt/TiO₂/(100)Si



PCT on
Pt/(100)MgO



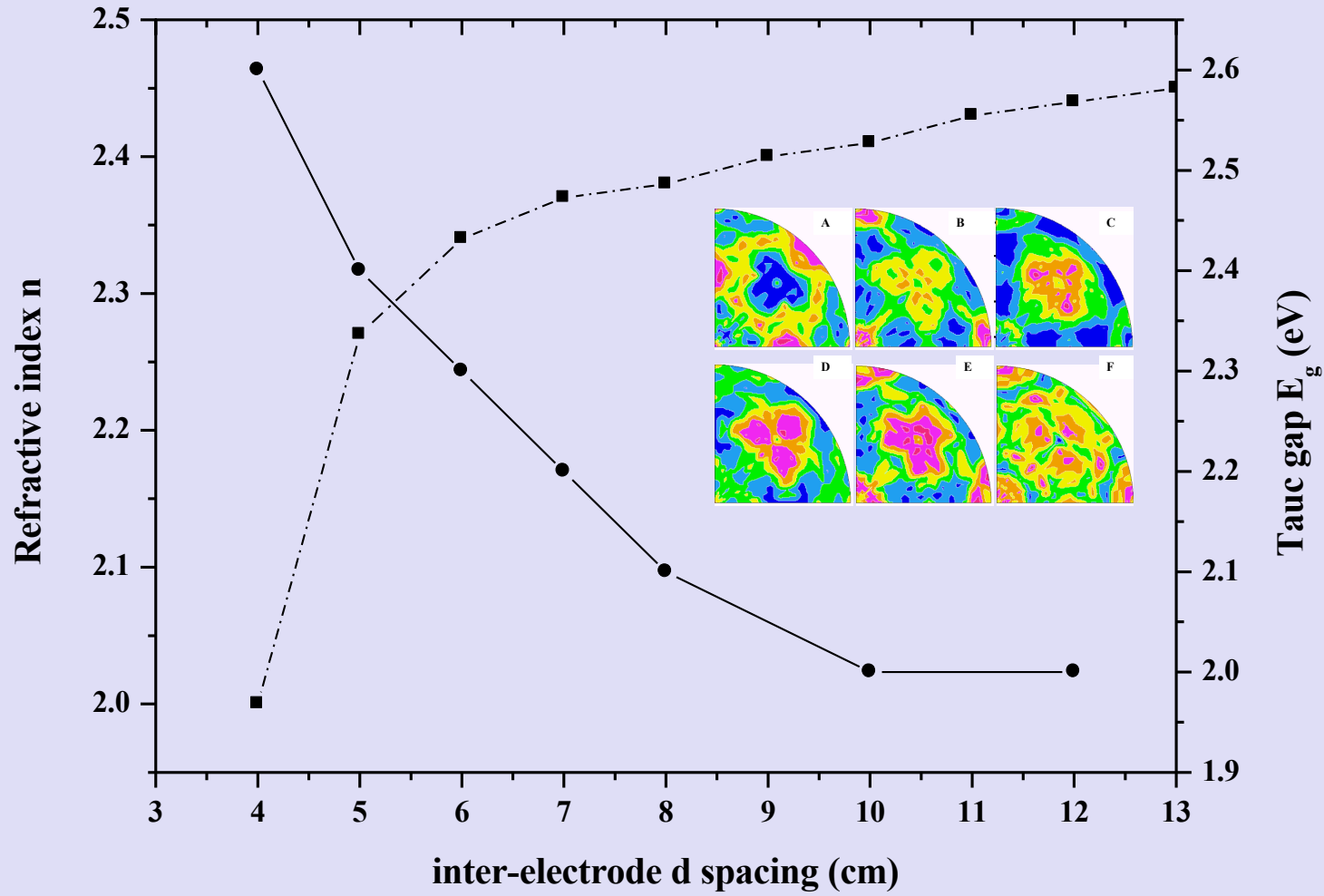
PCT on
Pt/(100)SrTiO₃



Texture Index m.r.d. ²	Pyroelectric Coefficient 10 ⁻⁸ C cm ⁻² K ⁻¹
2.1	0.3
5.1	1.5
7.9	1.1

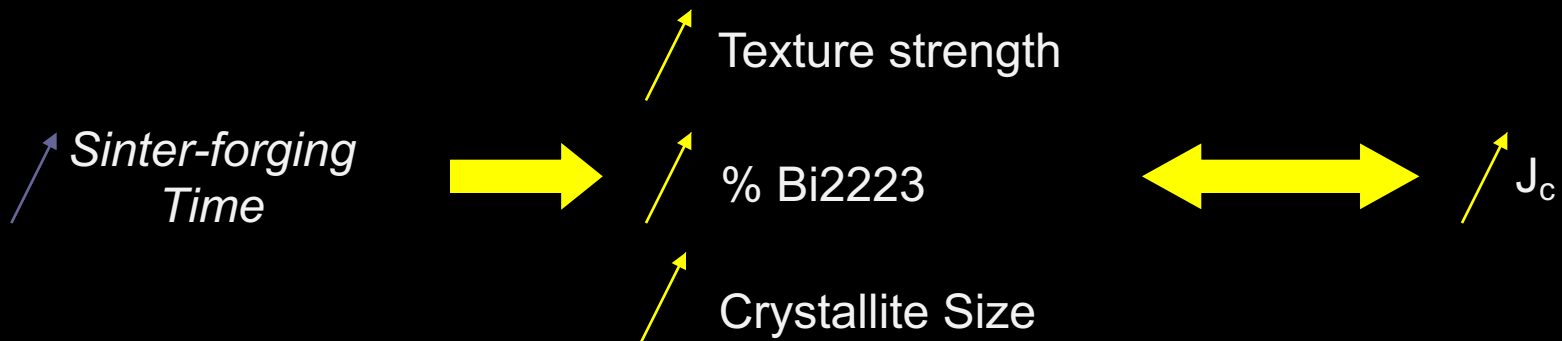
Enhancement of <001> texture

- Correlate macroscopic anisotropic properties: **Tauc gap in nano-Si**

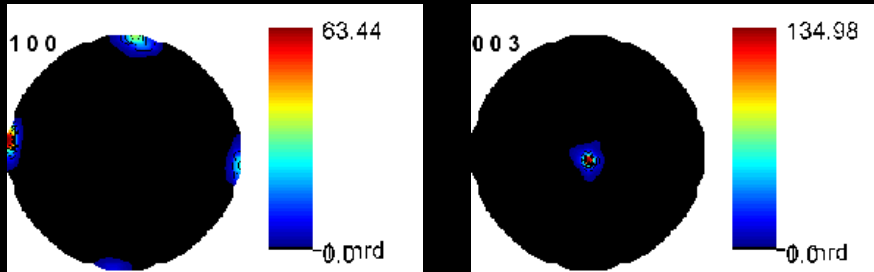


-Correlate macroscopic anisotropic properties: **Bi-2223 / Bi-2212** superconducting J_c 's

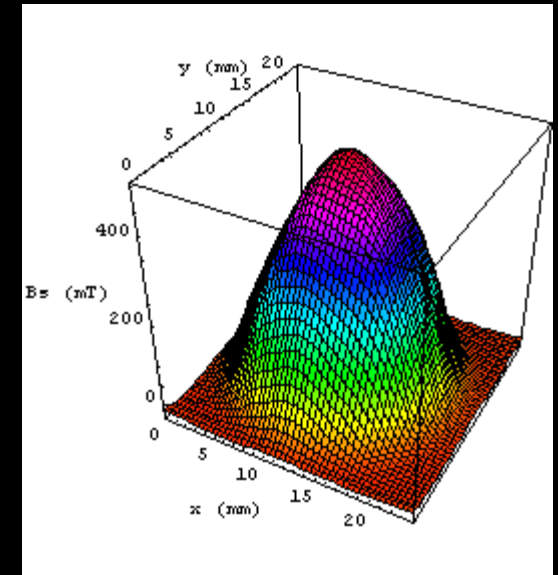
Sinter-forging dwell time (h)	Orientation Distribution Max (m.r.d.)		RP0 (%)	RP1 (%)	J_c (A/cm ²)
	Bi2212	Bi2223			
20	21.8	20.7	17.74	10.56	12500
50	24.1	24.4	17.05	11.04	15000
100	31.5	25.2	13.54	9.31	19000
150	65.4	27.2	16.24	12.25	20000



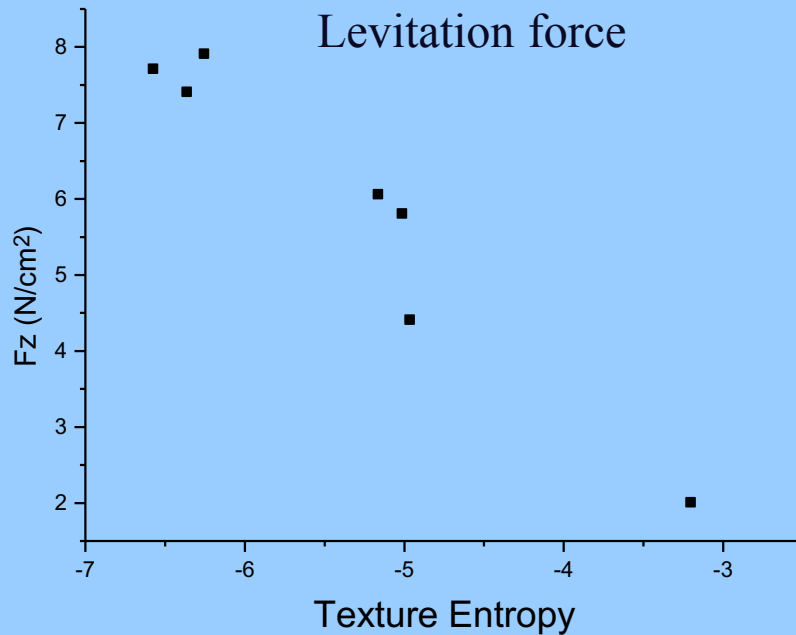
-Correlate macroscopic anisotropic properties: Levitation force and trapped flux in MTG-YBCO



Neutron pole figures (D1B-ILL)



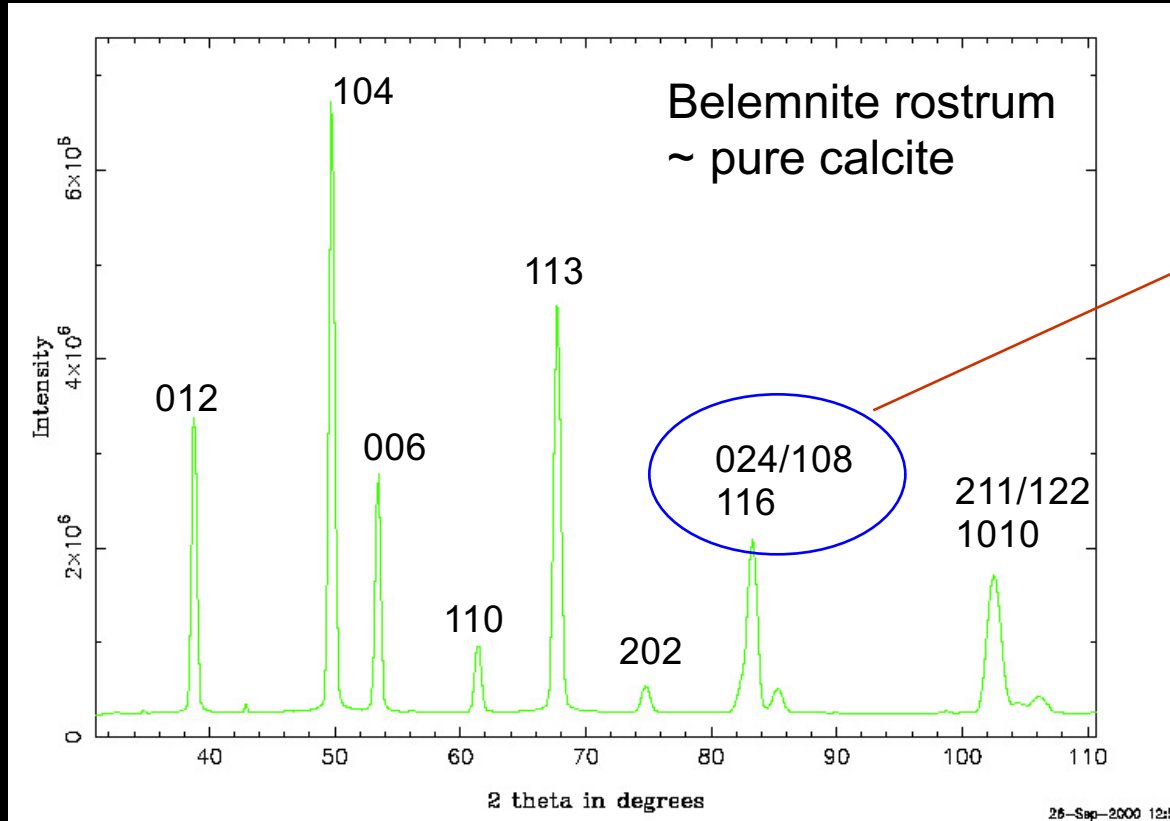
and trapped flux



Models ?

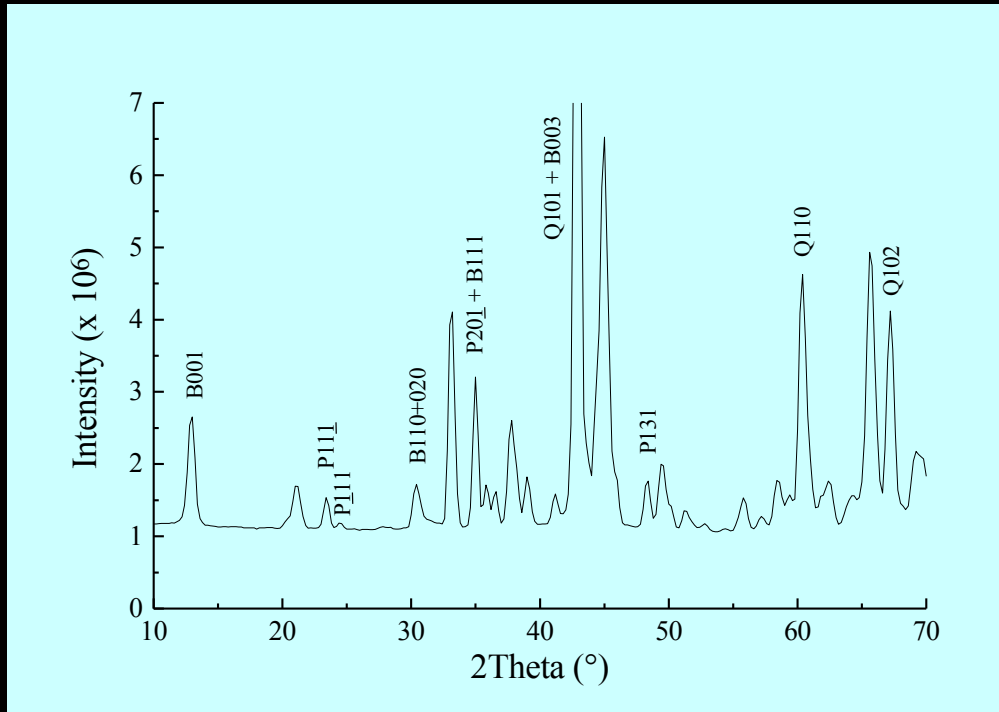
Why needing combined analysis

- Solve the peak-overlap problems (intra- and inter-phases)



Resolved during
ODF refinement

Polyphased Mylonite (Palm Canyon, CA)



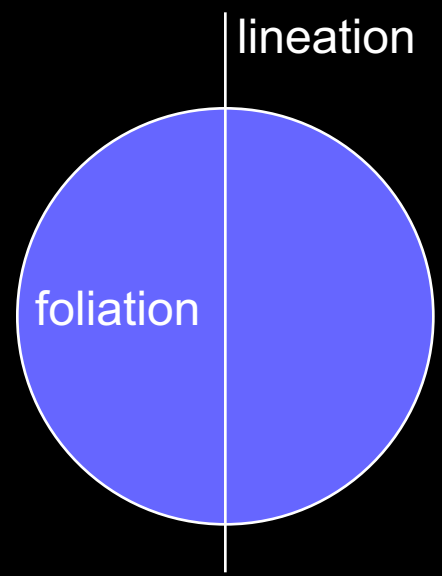
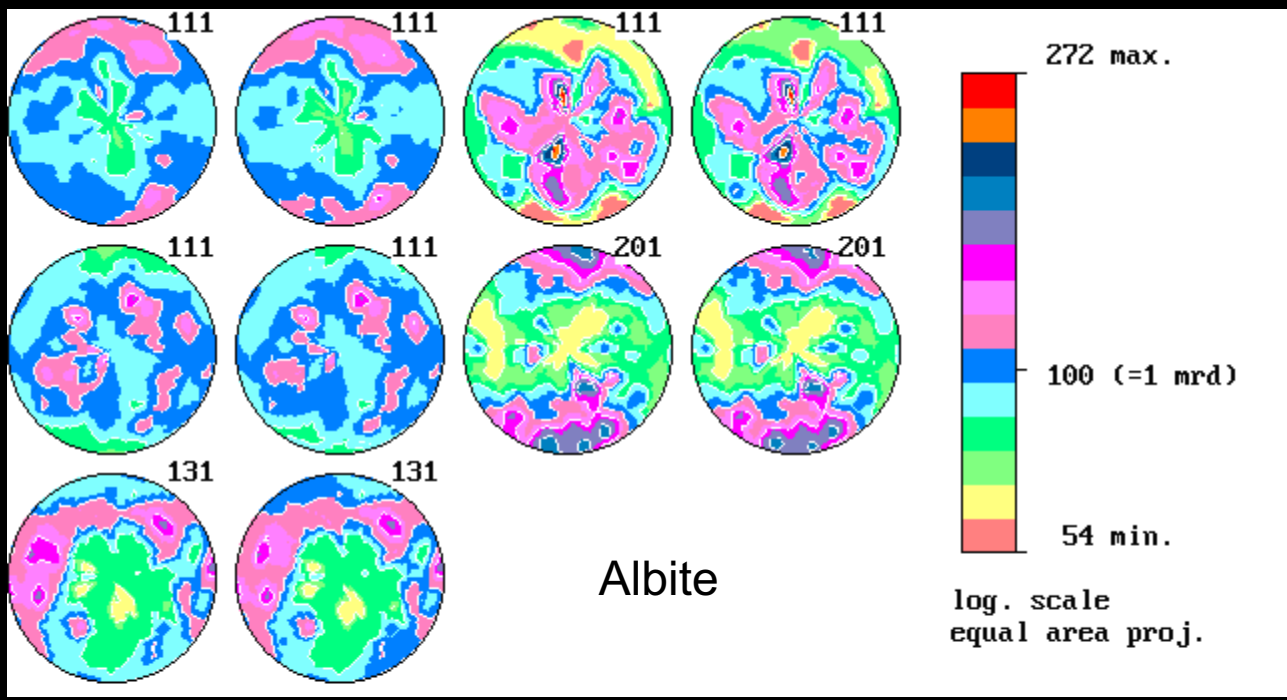
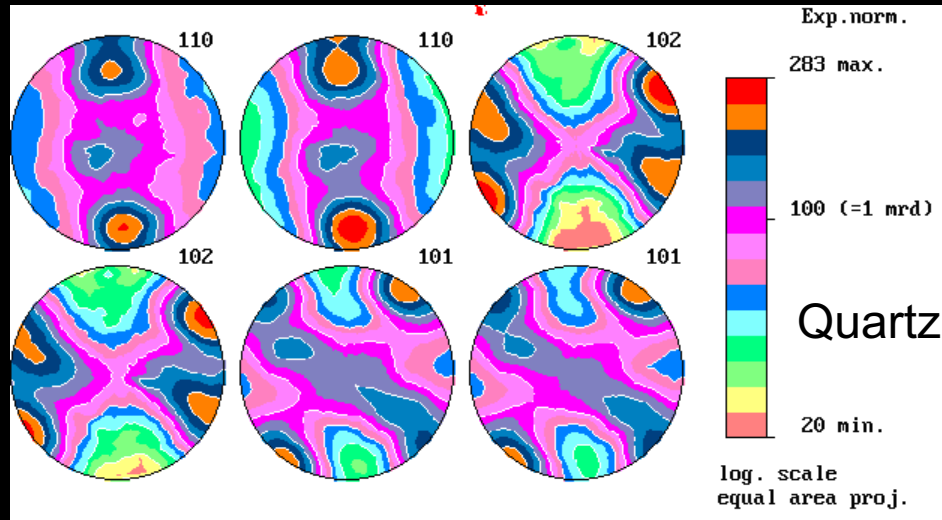
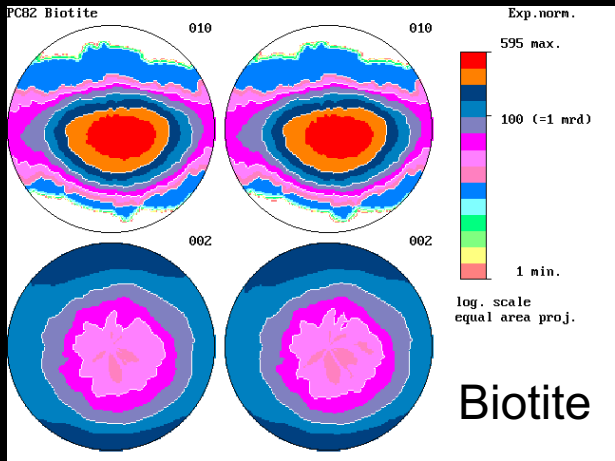
Using 0D detector
hardly manageable

Space group

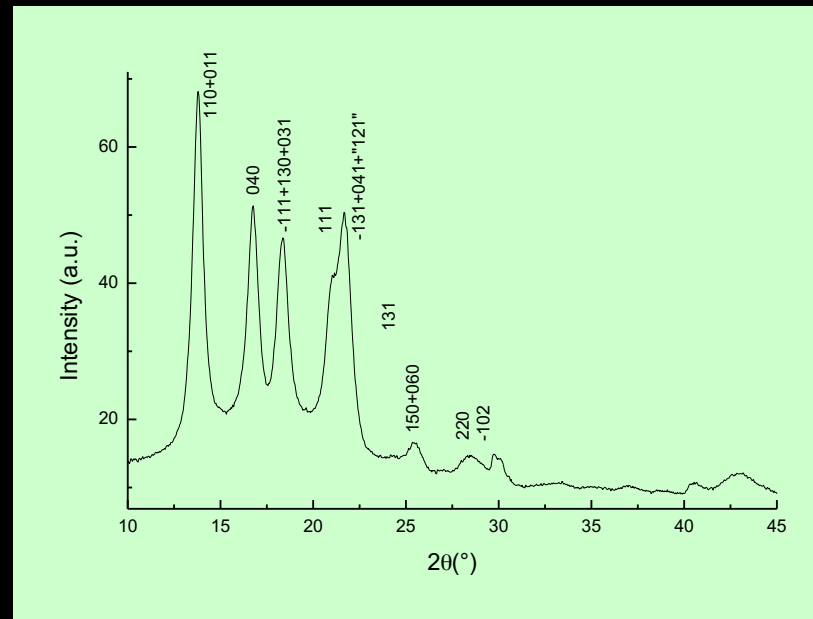
C2/m

R3

C-1



Plasma-treated polypropylene films

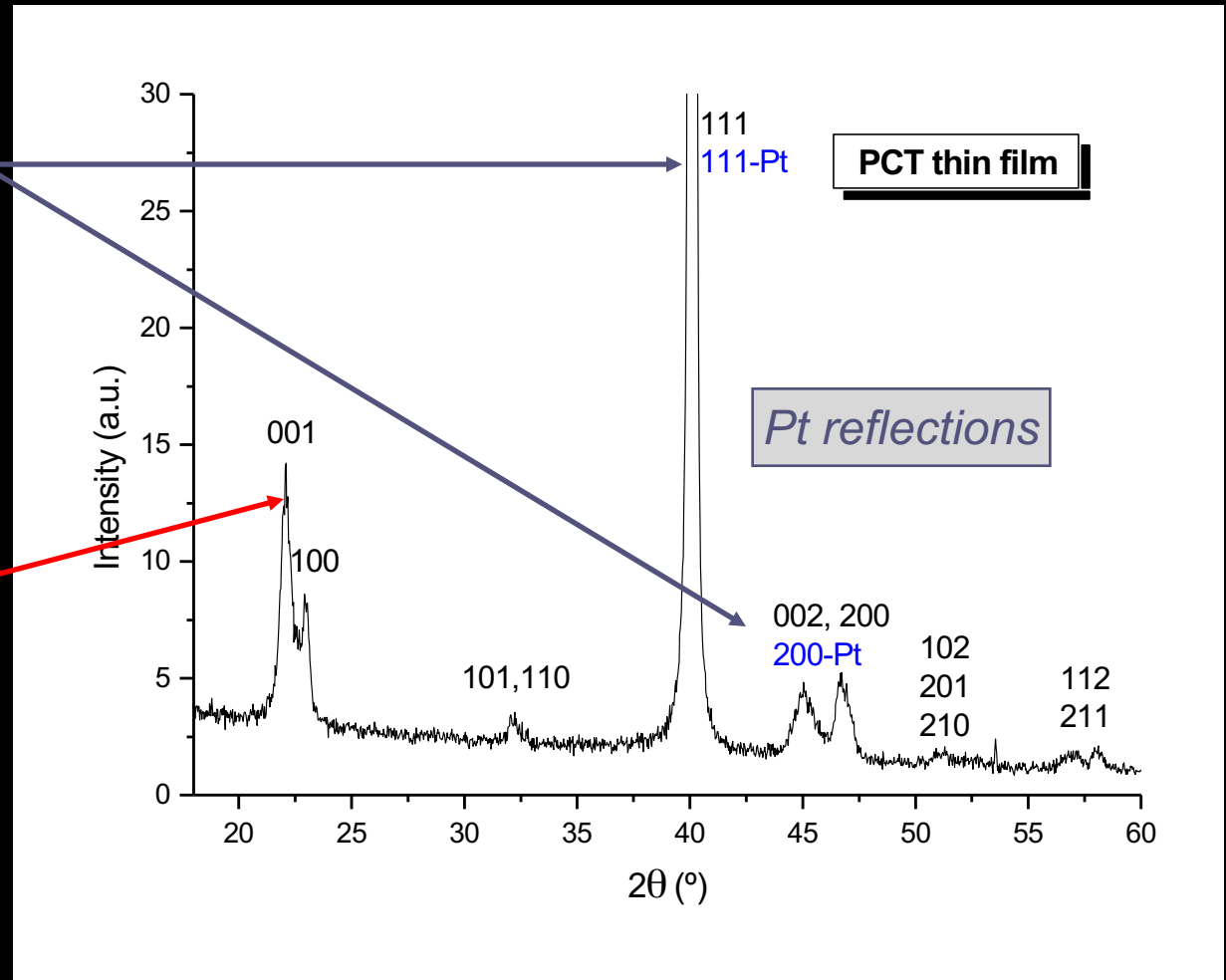


Large broadening + overlaps + amorphous phase

PCT ferroelectric films

Substrate influence:
Interphase overlaps of reflections from the film and the substrate

Intraphase overlaps



Minimum experimental requirements

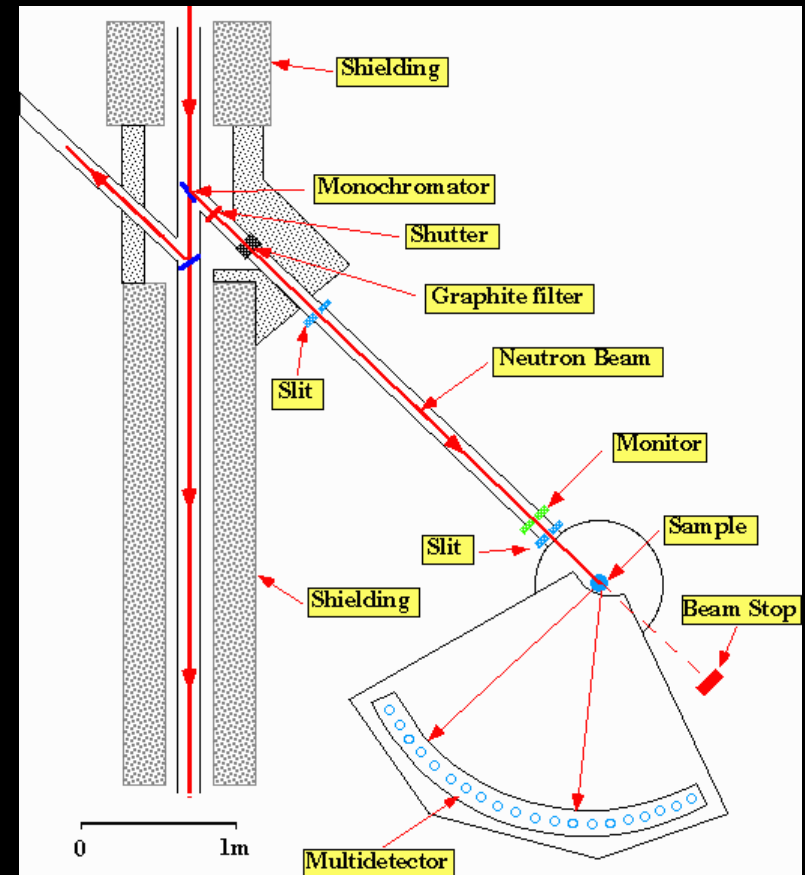
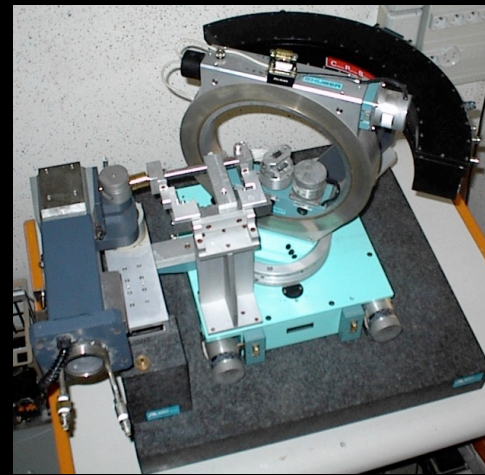
1D or 2D Detector + 4-circle diffractometer
(X-rays and neutrons)
CRISMAT, ILL

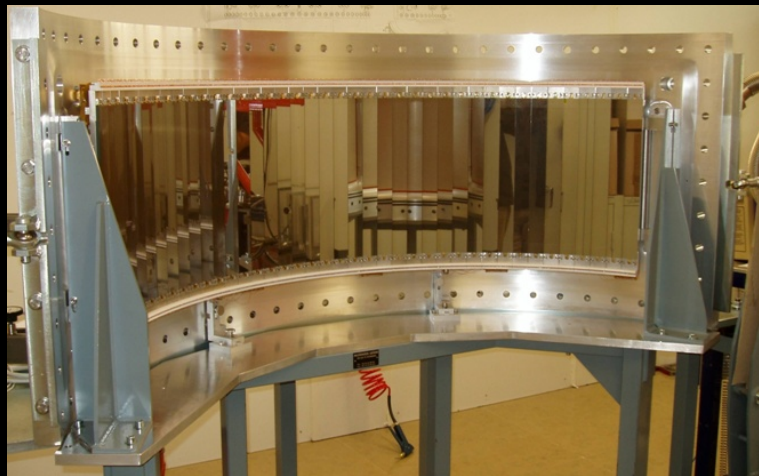
+

~1000 experiments (2 θ diagrams)
in as many sample orientations

+

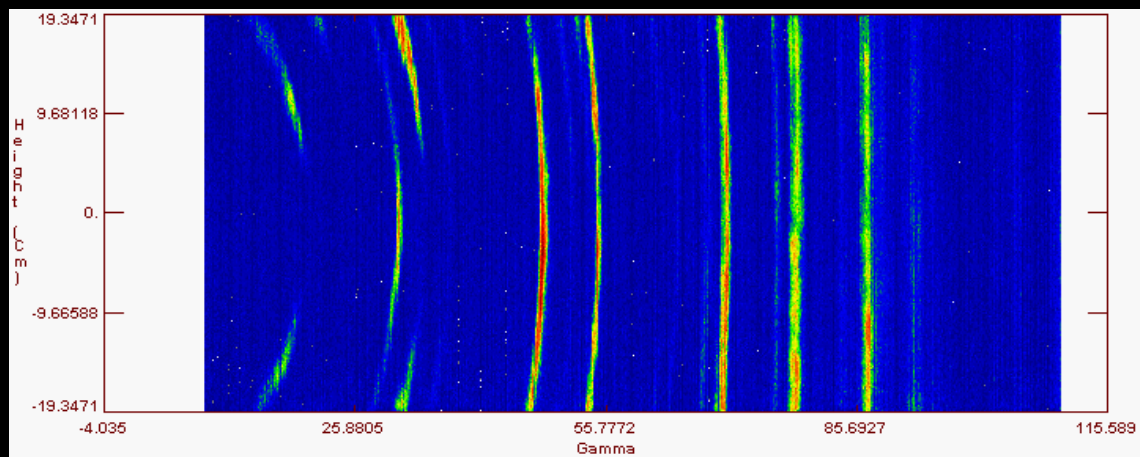
Instrument calibration
(peaks widths and shapes,
misalignments, defocusing ...)



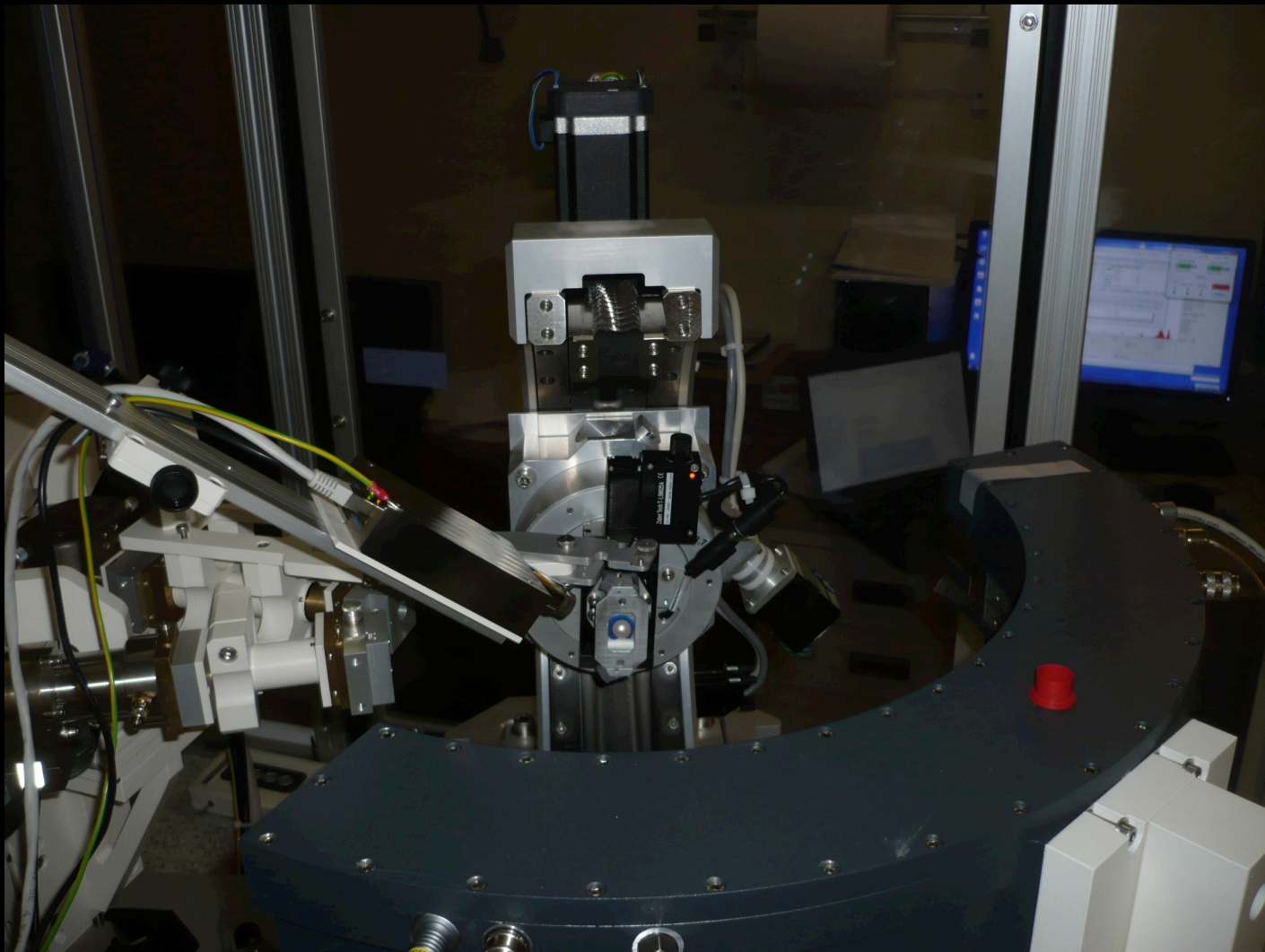


2D Curved Area Position Sensitive Detector

D19 - ILL



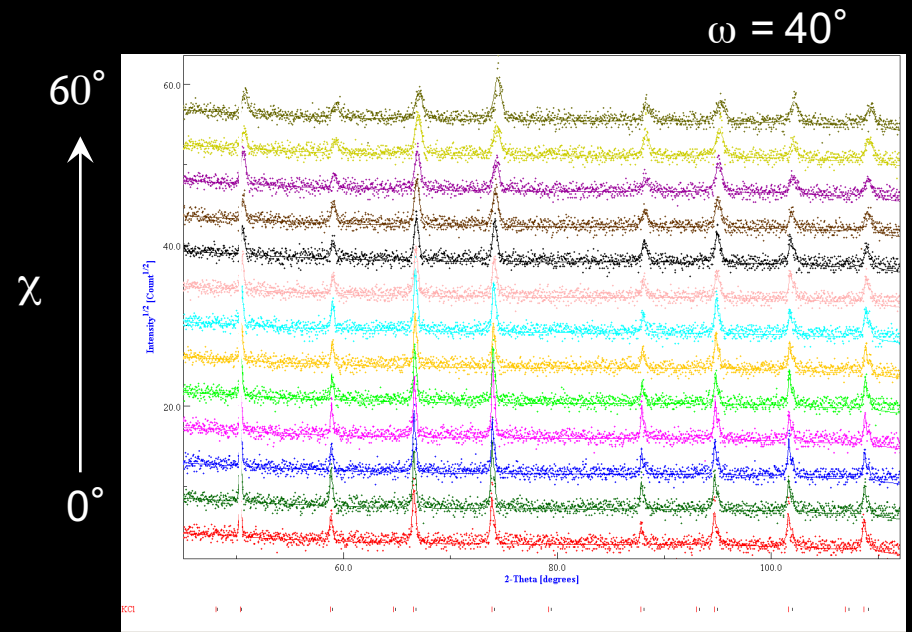
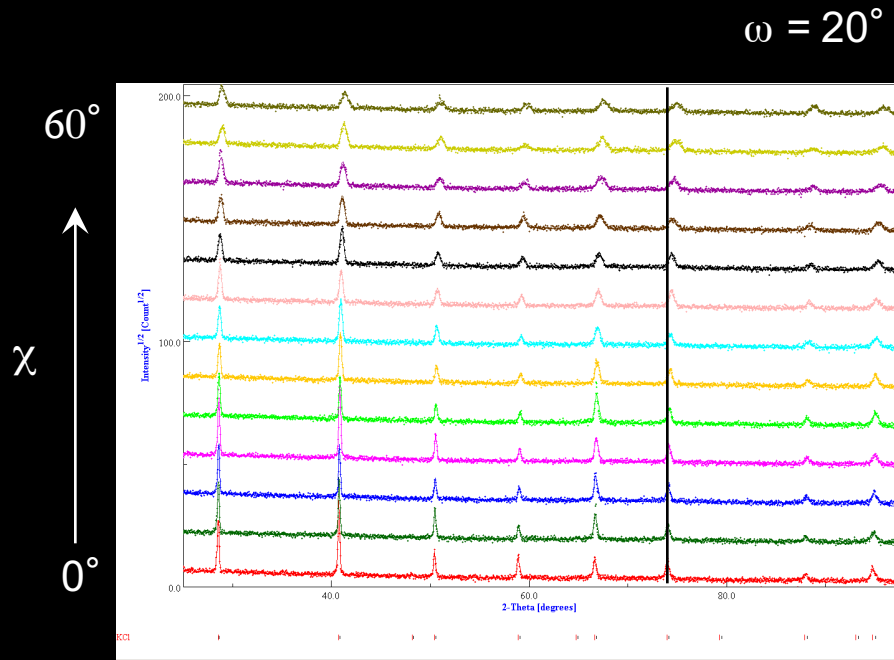
Belemnite sp.



1D CPS + 2 Ips + XRF

With 2 sources (Mo + Cu)

Calibration

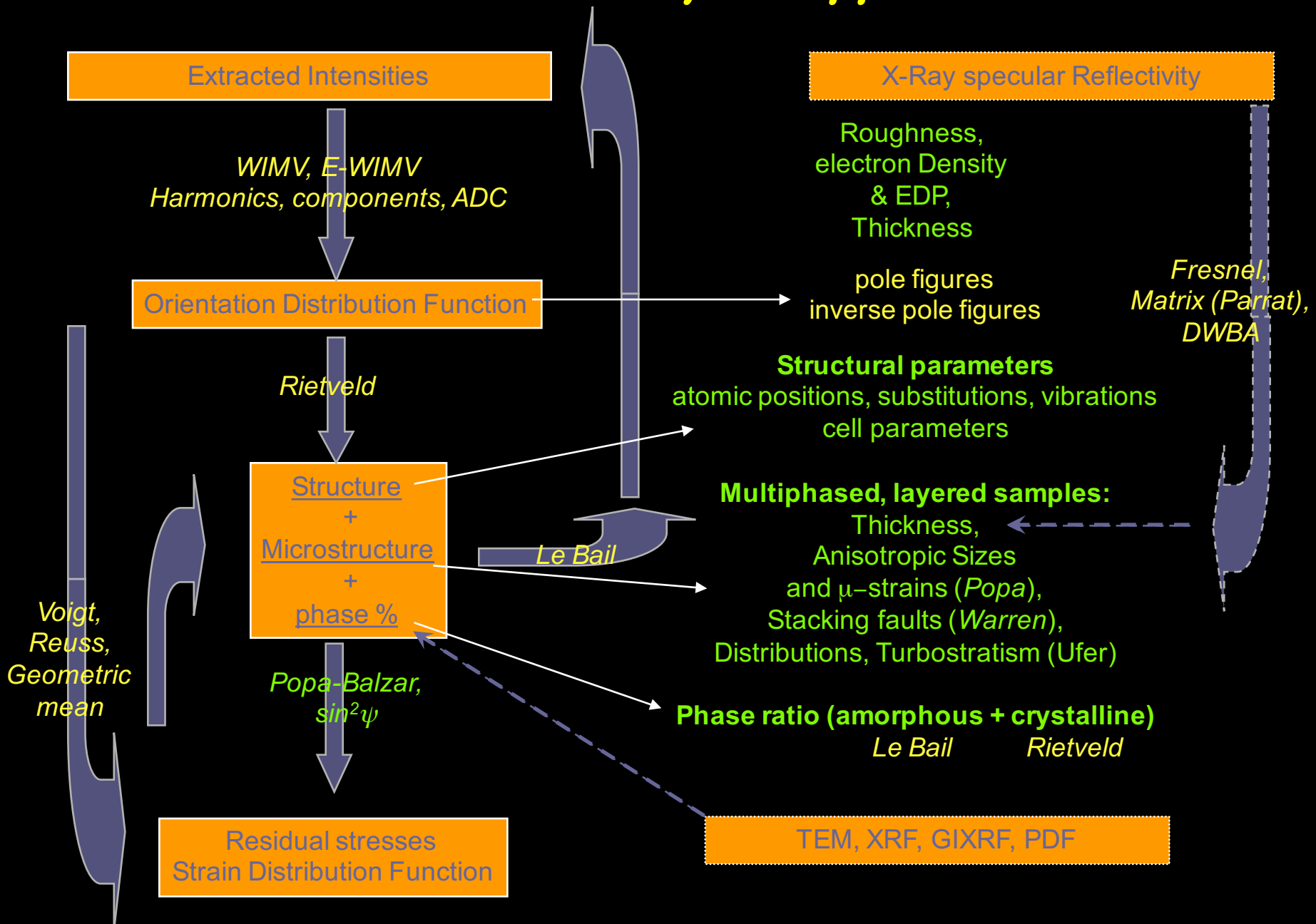


KCl, LaB₆ ...

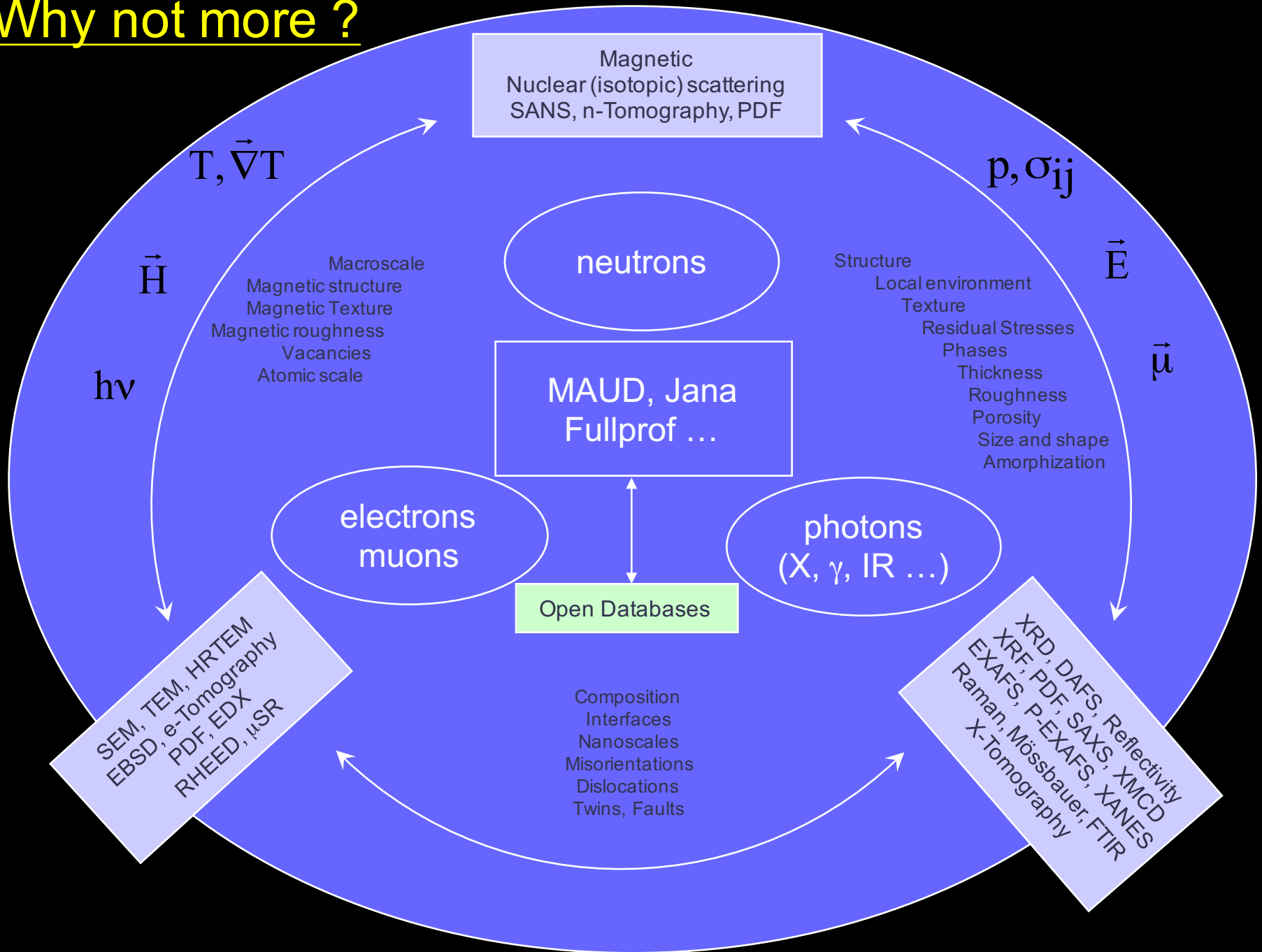


FWHM (ω , χ , 2θ ...)
2 θ shift
gaussianity
asymmetry
misalignments ...

Combined Analysis approach



Why not more ?



- Don't want or can't powderise your sample:
 - . Rare: Ice from deep cores, meteorite rocks ...
 - . Expensive: high-tech materials
 - . Impossible: hard materials, polymers, thin structures ...
- Decreases instrument time:
 - . $5^\circ \times 5^\circ$ grid = 1368 points / pole figure
 - . ODF: needs as much pole figures as possible
- Access to other parameters:
 - . crystal sizes, micro-strains, stacking faults + twins (QMA)
 - . residual strains and stresses (QSA)
 - . Structure determination
 - . Phase proportions (QPA)
 - . Thicknesses, roughnesses (XRR)

- Avoid false minima due to parameter correlation:
 - . phase and texture
 - . Structure and texture
 - . Structure and strains
 - . Thickness and phase
 - ...

- Benefit of these correlation to access "true" values
 - Textured materials: between powder and single-crystal,
angular discrimination

- Easier to practice !