

# X-RAY DIFFRACTION : an overview of the technique

Dr. M. Morales, Lab. CIMAP - Ensicaen

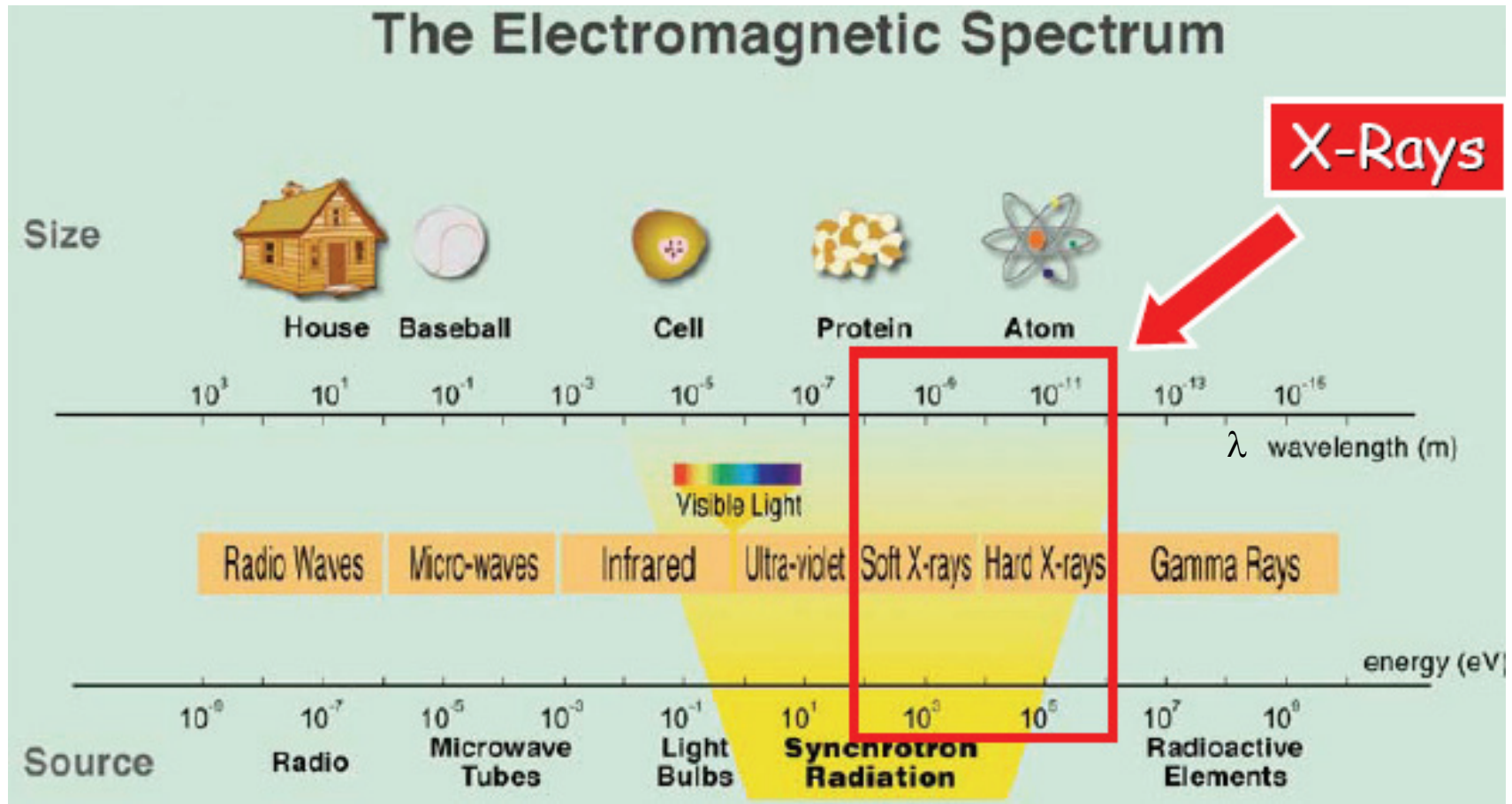
[magali.morales@ensicaen.fr](mailto:magali.morales@ensicaen.fr)

CiMap

## OUTLINE :

- \* Electromagnetic properties of X-rays
  - \* Production of X-rays
  - \* Interactions of X-rays with matter
- \* X-ray diffraction : investigation of the fine structure of the matter
  - \* Some basics of crystallography
  - \* X-ray diffraction by crystals
- \* Summary

**X-rays = Electromagnetic (EM) radiations lying between Ultraviolet light and Gamma rays in the electromagnetic spectrum**

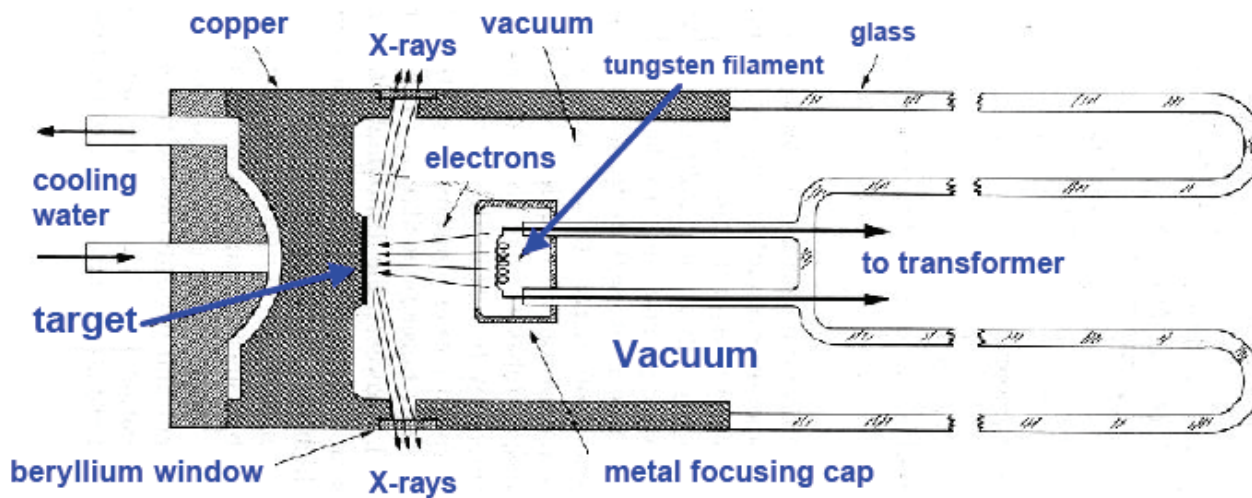


↪ Associated particle = photon (no charge, no mass) ; photon energy  $E = \frac{hc}{\lambda}$

↪ X-rays are characterized by relatively short wavelength :  $\lambda = 0.01-100 \text{ \AA}$  ( $10^{-10}-10^{-6} \text{ cm}$ ), with Hard X-Ray on one hand and Soft X-Ray on the other.

# X-ray production = X-ray tube

X-rays are conventionally produced by either **conversion of kinetic energy of charged particles into radiation (continuous X-ray spectrum)** or by **atom excitation in a metal target (Cu, Al, Mo, Mg) upon which fast moving electrons impinge**.

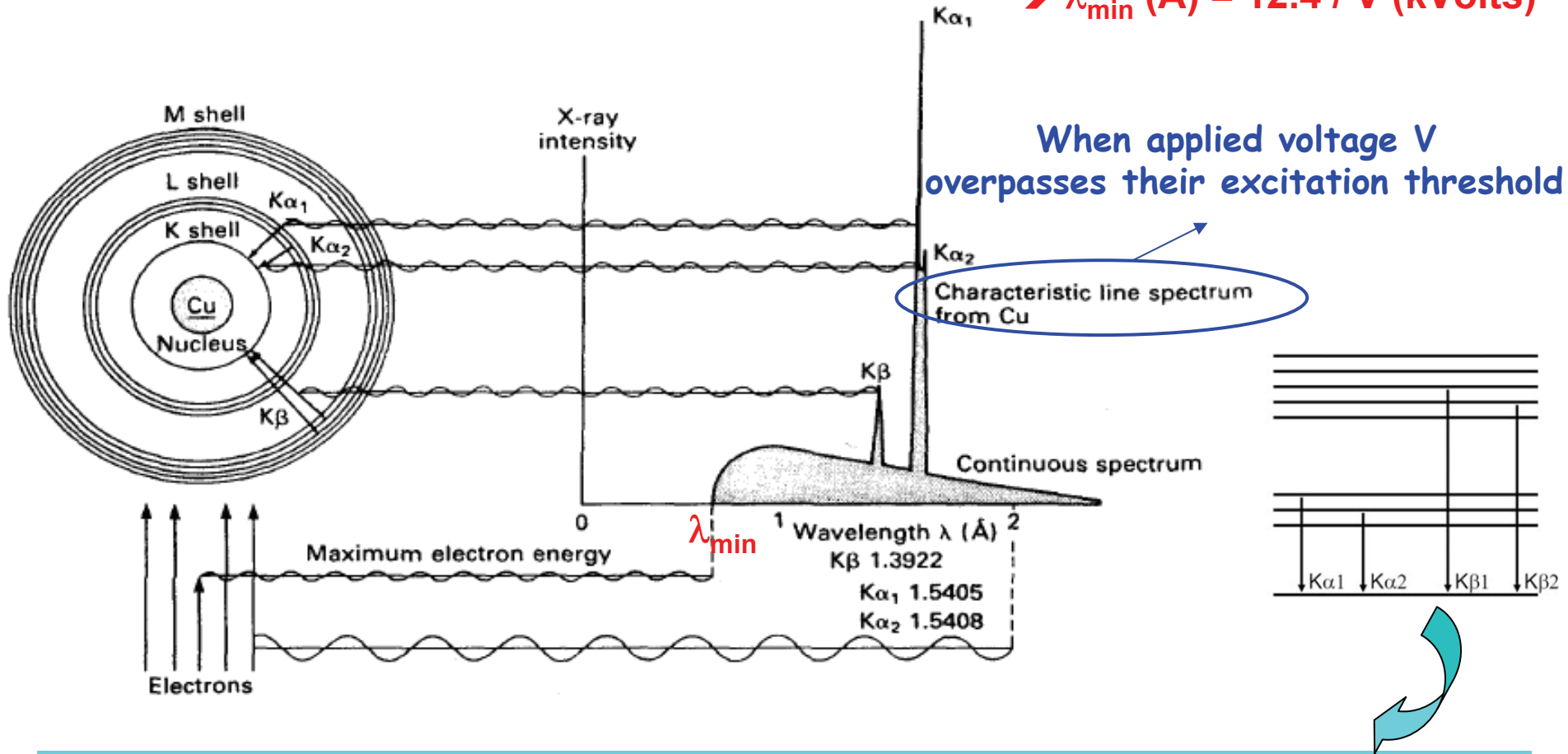


Sealed-off filament  
X-ray tube

- Anode is a water-cooled Cu block containing the desired target
- **Source of electrons = hot W filament** (Joule effect,  $I \sim 40 \text{ mA}$ ), accelerated by application of high accelerating voltage  $V$  ( $\sim 45 \text{ kV}$ ) between cathode (W) and anode (target).

**Continuous spectrum generation = electron deceleration in the target  
(complete or partial electrons collision)**

$\rightarrow \lambda_{\min} (\text{\AA}) = 12.4 / V \text{ (kVolts)}$



**Bombarding electrons can eject electrons from the metal target atom inner shells: vacancies will be quickly filled by electrons dropping down from higher levels, emitting x-rays with sharply defined frequencies associated with the difference between the atomic energy levels of the target atoms.**

Refractive index of X-rays :  $n = 1 - \delta - i\beta$

$$\delta = \frac{r_e}{2\pi} \lambda^2 \rho_e$$

dispersion

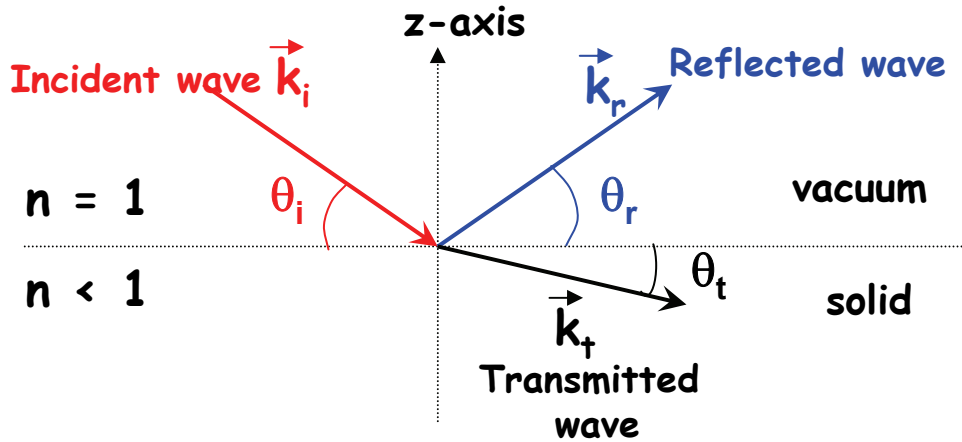
$$\beta = \frac{\lambda\mu}{4\pi}$$

absorption

$r_e$ : classical radius of the electron ( $0.28 \cdot 10^{-5}$  nm)

$\rho_e$ : electron density of the material

$\mu$ : linear absorption coefficient



Typically  $\delta$ :  $10^{-5}$  to  $10^{-6}$ , and  $\beta \sim \delta/10$ ;

$\delta > 0$ , i.e.  $n < 1$  but slightly  
 → total reflection possible if :

$$\cos \theta_c = n = 1 - \delta \approx 1 - \frac{\theta_c^2}{2}$$

↪ At low angle ( $\sim 0.1^\circ$  to  $1^\circ$ ), X-ray beam **totally reflected** and propagation in material is evanescent (penetration depth : 2 to 3 nm) : **surface study + reflectivity**

↪ At higher angle : **penetration depth of some tens of  $\mu\text{m}$  thickness !**  
 As  $n < 1$  but slightly : X-ray linear propagation in the matter.

# Interaction of X-rays with matter ( $\theta \gg 1^\circ$ )

X-rays mostly interact with atomic electrons → 3 options:

1) Penetrate

2) Scatter = change in direction of electromagnetic waves with or not an energy exchange between photons and matter

3) Be absorbed (partially or totally)

## 5 basic interactions of X-rays with matter:

(different competing processes that are involved simultaneously)

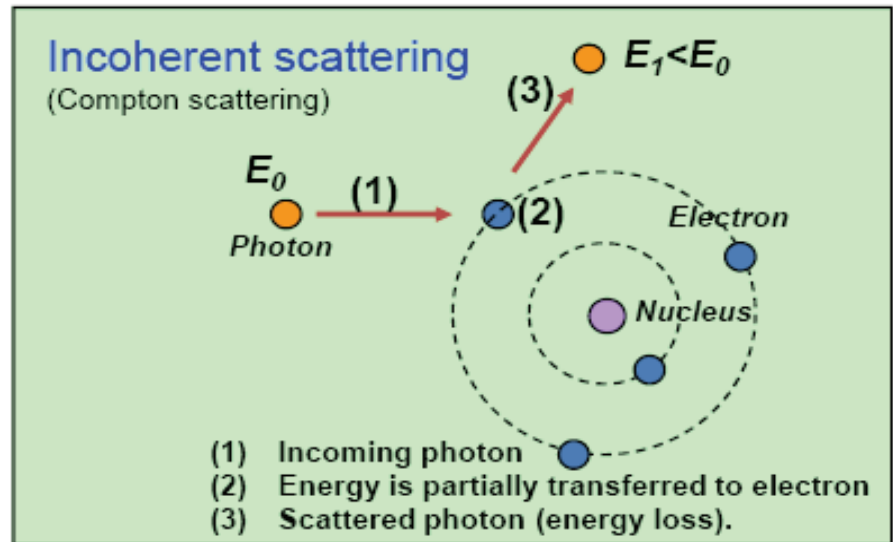
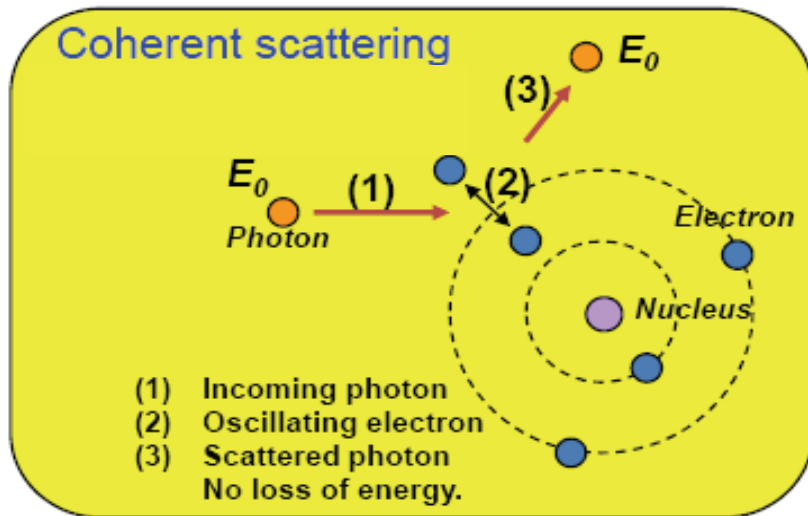
- Coherent scattering : Thomson diffusion (elastic) if  $h\nu < E_{\text{binding}}$
- Incoherent scattering : Compton diffusion (inelastic) if  $h\nu \gg E_{\text{binding}}$
- Photoelectric absorption effect if  $h\nu \geq E_{\text{binding}}$
- Pair production (electron - positron pair) :  $E > 1 \text{ MeV}$
- Photodisintegration  $E > 10 \text{ MeV}$

For 10 eV - 30 KeV :

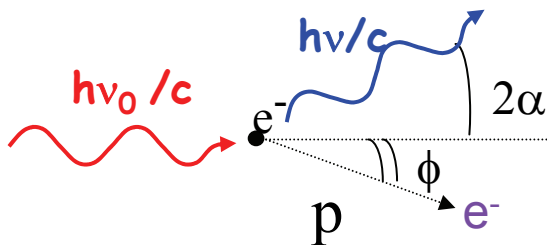
pair production and photodisintegration are negligible

## Interactions of X-rays with matter:

- **Coherent scattering** : Thomson diffusion (elastic) if  $h\nu < E_{\text{binding}}$  (at low energy : Rayleigh diffusion)
- **Incoherent scattering** : Compton diffusion (inelastic) if  $h\nu \gg E_{\text{binding}}$  (at low energy : Raman diffusion)



### Compton scattering energy loss :

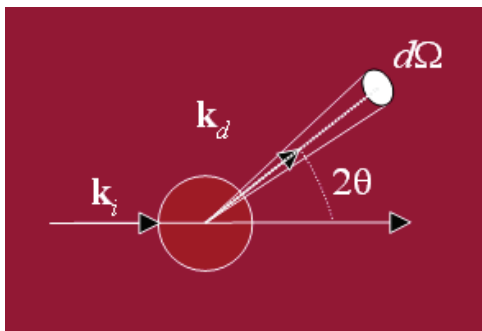
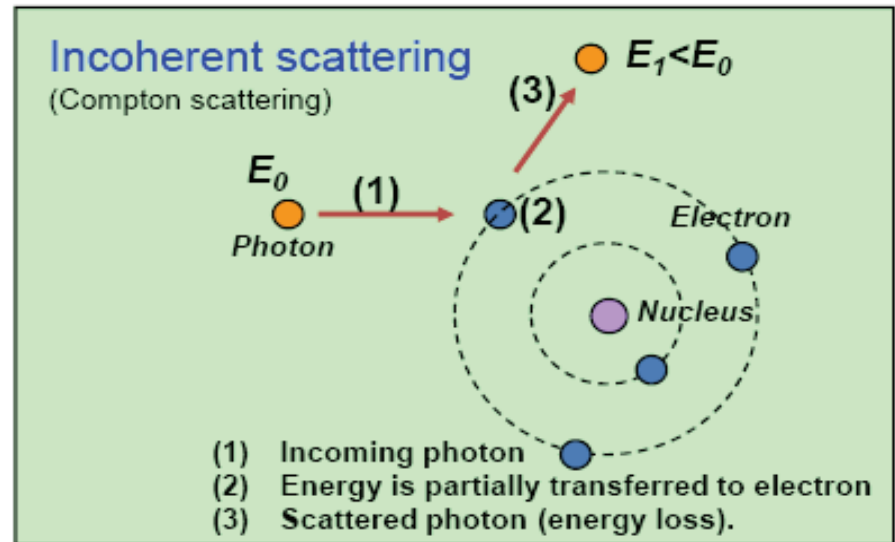
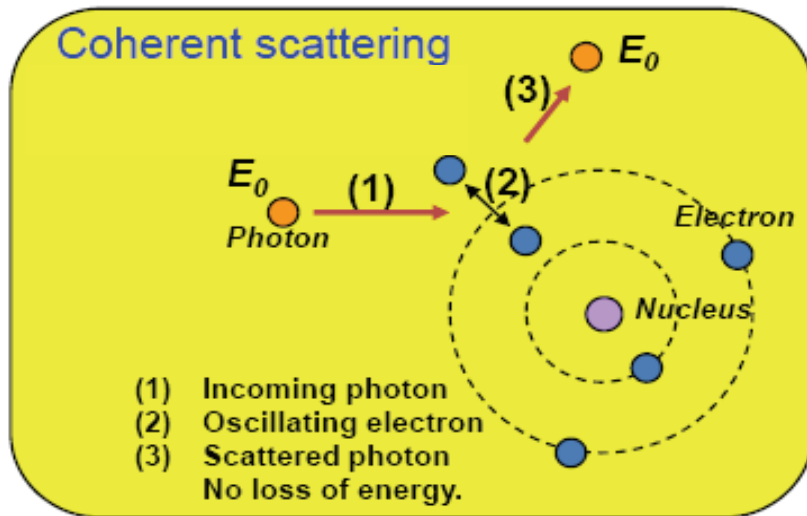


$$\lambda - \lambda_0 = \Delta\lambda = \frac{h}{m_0c} (1 - \cos(2\alpha)) = 0.0242 (1 - \cos(2\alpha))$$



## Interactions of X-rays with matter:

- Coherent scattering : Thomson diffusion (elastic) if  $h\nu < E_{\text{binding}}$  (at low energy : Rayleigh diffusion)
- Incoherent scattering : Compton diffusion (inelastic) if  $h\nu \gg E_{\text{binding}}$  (at low energy : Raman diffusion)



- Some definitions :
- $\vec{k}_i$  and  $\vec{k}_d$  : incident and diffuse scattering vectors
- $2\theta$  = diffusion angle
- Elastic scattering :  $k_i = k_d$
- Inelastic scattering  $k_i \neq k_d$

# X - ray absorption

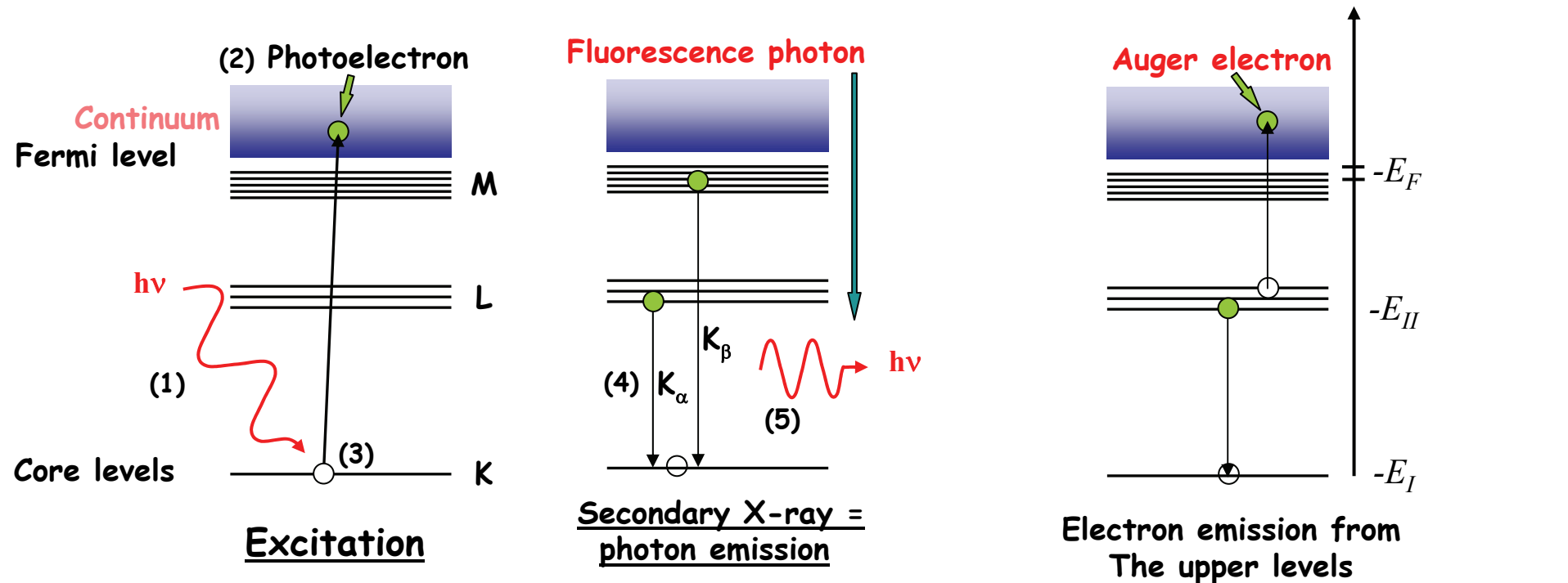
For  $E < 1000$  keV photoelectric effect is dominant

- Photoelectric effect = photon absorption interaction

- Absorbed photon if  $h\nu > E_B$  ( $E_B$  electron binding energy)

- Excitation : emitted photoelectron ( $E = h\nu - E_B - \Phi$ )

- Desexcitation mechanisms : fluorescence photon ( $h\nu = E_I - E_{II}$ )  
or Auger electron ( $E = E_I - E_{II} - E_{III}$ )

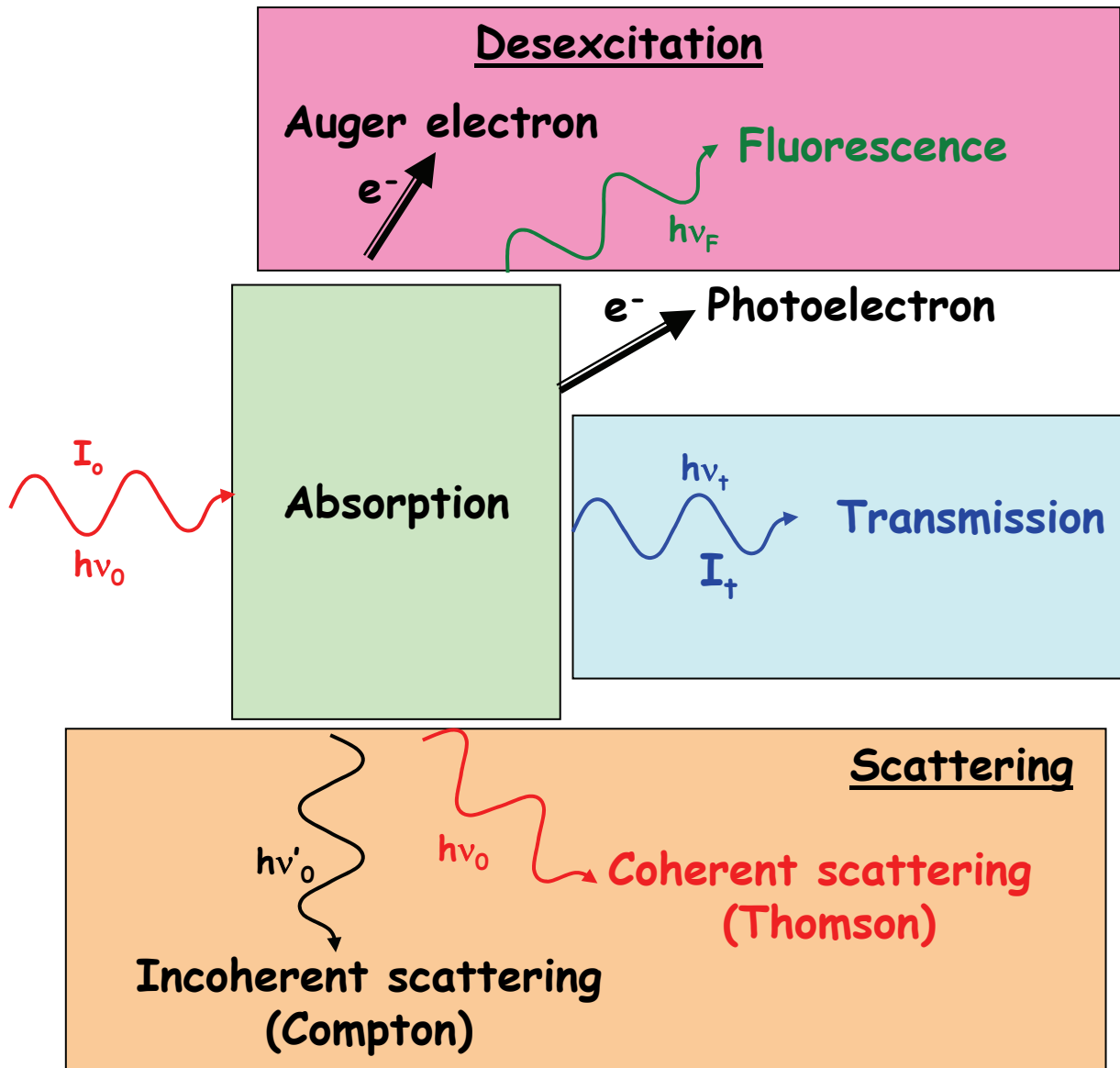


(1) Incoming photon  
(2) Expelled electron  
(3) Hole is created in shell

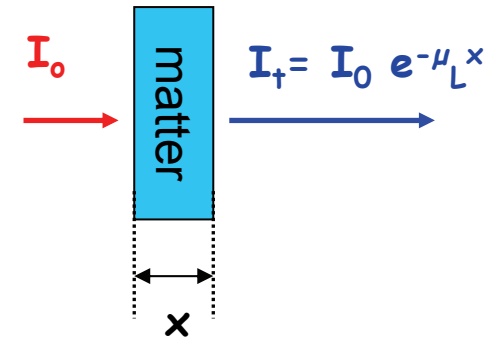
(4) Outer shell electron moves to inner shell hole  
(5) Energy excess emitted as photon.

(4') Hole is occupied by outer  $e^-$   
(5') Excitation energy transferred to  $e^-$   
(6') Electron ejected from atom

# Interaction of X-rays with matter



## Absorption



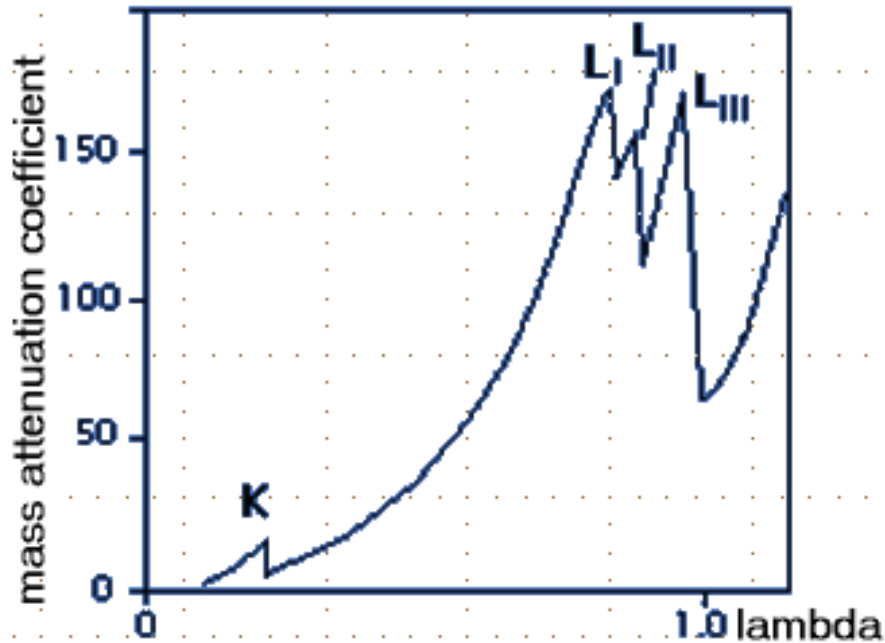
$\mu_L$  = linear absorption

$$\text{coefficient} = \left( \frac{\mu_L}{\rho} \right) \rho = \mu \rho$$

with  $\rho$  = volumic mass ( $\text{g}\cdot\text{cm}^{-3}$ ) and  $\mu$  = mass absorption coefficient in  $\text{g}^{-1}\cdot\text{cm}^{-2}$  (tabulated in the International Tables of Crystallography)

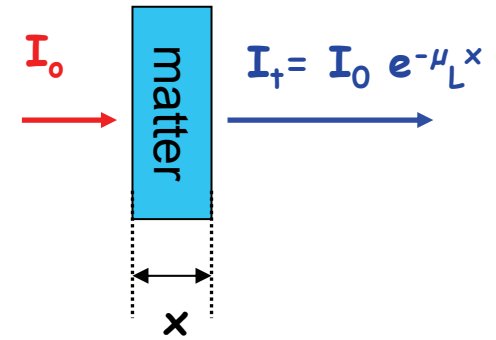
$$I_t = I_0 e^{-\mu \rho x}$$

# Interaction of X-rays with matter



↪ Attenuation globally increases with  $\lambda$  and for a given  $\lambda$ ,  $\mu$  increase with  $Z$ .

↪ Absorption discontinuities (K, L<sub>I</sub>, L<sub>II</sub>, L<sub>III</sub>) can be evidenced : they appear when energy of incident photon,  $W = h.c/\lambda \geq W_K$



$\mu_L$  = linear absorption

$$\text{coefficient} = \left( \frac{\mu_L}{\rho} \right) \rho = \mu \rho$$

with  $\rho$  = volumic mass ( $\text{g.cm}^{-3}$ ) and  $\mu$  = mass absorption coefficient in  $\text{g}^{-1}.\text{cm}^{-2}$  (tabulated in the International Tables of Crystallography)

$$I_t = I_0 e^{-\mu \rho x}$$

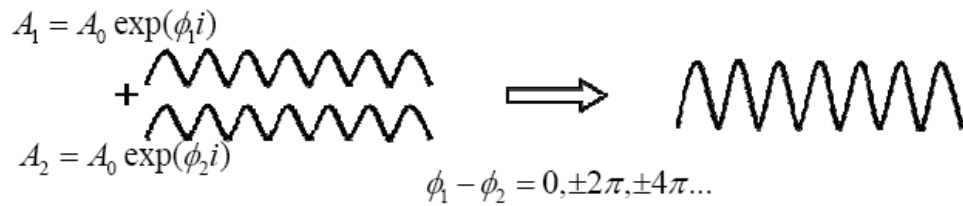
# X-ray diffraction

↪ Diffraction is the effect following the interaction of an EM wave with an object having size dimension comparable with the period of the wave.

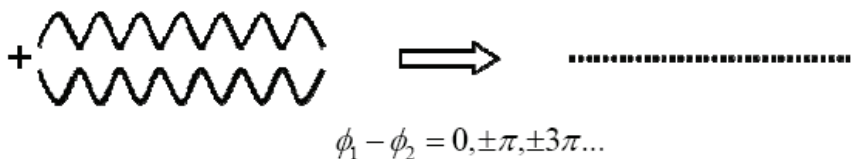
↪ X-rays have wavelengths on the order of a few angstroms, the same as typical interatomic distances in crystalline solids = tool for investigation of fine structure of matter



X-ray diffraction is based on elastic scattering and results from the coherent sum of all EM waves that are diffused from each atom of a periodic structure constituting the matter (the same that occurs with visible light interaction with a grating).



Coherent sum :  
Constructive interference



Non coherent sum:  
Destructive interference

# Interaction of X-rays with matter : associated techniques

## EMISSION :

- \* X-rays : Fluorescence (chemical analysis)
- \* Electrons:
  - Photoelectrons, Auger electrons (analysis)
  - Photoelectrons diffraction (local structure)
  - Photoemission (band structure)

## REFRACTION :

Reflectivity (surface, interface)  
Surface diffraction (surfaces)  
Stationnary waves (surfaces)

ABSORPTION : XAS, EXAFS, XANES (local order); Dichroïsm (Magnetism, surfaces)

Incident X-Rays

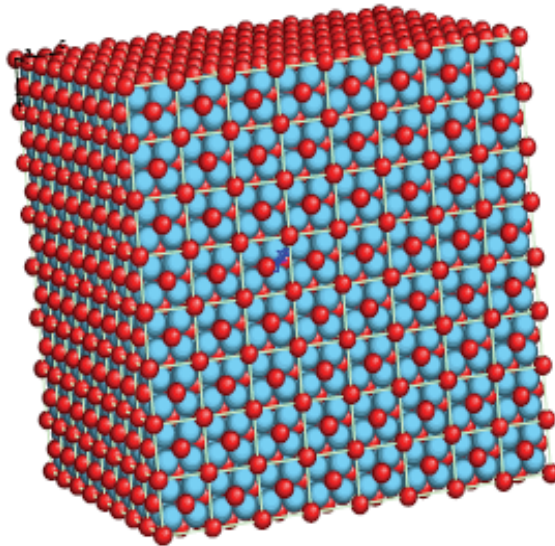
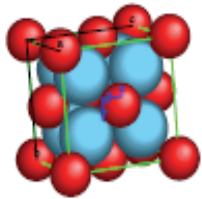
Matter

## DIFFUSION:

**Diffraction (Structures):** diffuse diffusion (Disorder, liquids)  
Compton diffusion (Electronic structure )  
Small angles diffusion (Polymers, liquid crystals, agregates)  
Magnetic, inelastic , coherent diffusion... (synchrotrons)

# Now we need some basics of crystallography !

↳ Crystallised solid : 3D periodic array of atoms

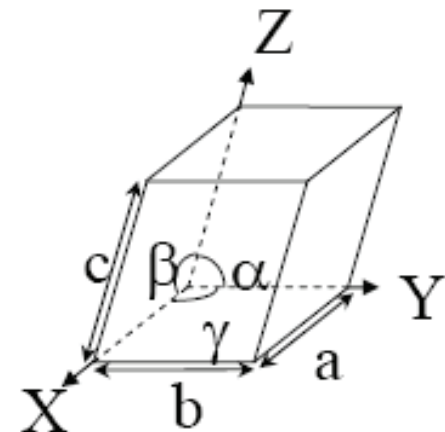


Macroscopic crystal =  
crystal lattice

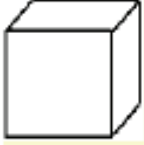

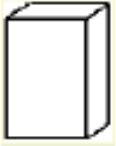
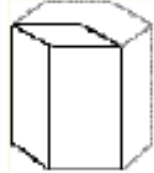
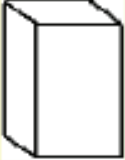

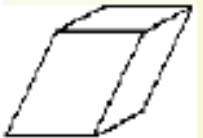
Unit cell :  
The smallest  
set of atoms  
arranged in a  
particular way.

periodically repeated  
in three dimensions

In 3D, the unit cell =  
parallelepiped built on the crystallographic  
basis vectors  $a$ ,  $b$ ,  $c$  of the direct lattice,  
is determined by 6 parameters:  $a$ ,  $b$  and  $c$   
cell lengths and the  $\alpha$ ,  $\beta$  and  $\gamma$  angles  
which can be determined by XRD



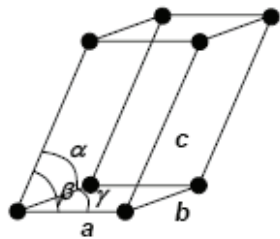
# Cristalline structures classification (function of their symmetry) → 3D Bravais lattice systems (7)

<b>cubic</b>		$a = b = c$ $\alpha = \beta = \gamma = 90^\circ$
<b>tetragonal</b>		$a = b \neq c$ $\alpha = \beta = \gamma = 90^\circ$
<b>orthorhombic</b>		$a \neq b \neq c$ $\alpha = \beta = \gamma = 90^\circ$
<b>hexagonal</b>		$a = b \neq c$ $\alpha = \beta = 90^\circ$ $\gamma = 120^\circ$
<b>monoclinic</b>		$a \neq b \neq c$ $\alpha = \gamma = 90^\circ$ $\beta \neq 90^\circ$
<b>rhombohedral</b>		$a = b = c$ $\alpha = \beta = \gamma \neq 90^\circ$
<b>triclinic</b>		$a \neq b \neq c$ $\alpha \neq \beta \neq \gamma \neq 90^\circ$

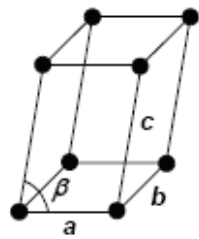


# Seven crystal systems + four lattice centering types = 14 Bravais lattices

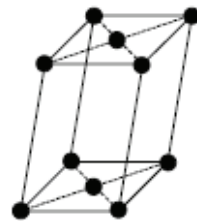
## 14 Bravais lattices $\Lambda(r)$



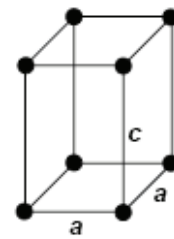
Triclinic P



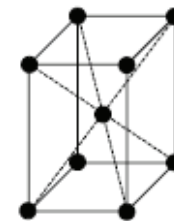
Monoclinic P



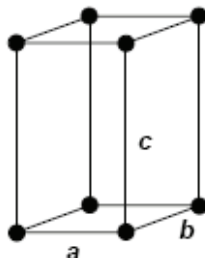
Monoclinic C



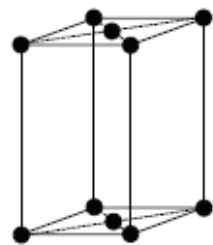
Tetragonal P



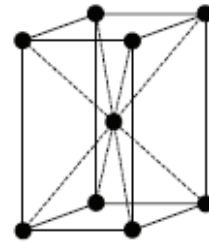
Tetragonal I



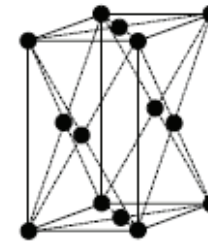
Orthorhombic P



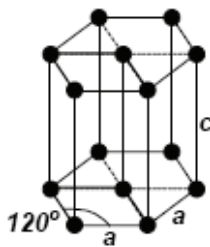
Orthorhombic C



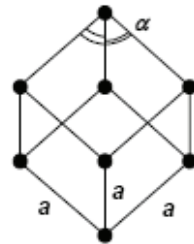
Orthorhombic I



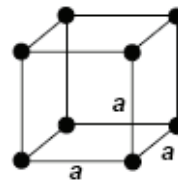
Orthorhombic F



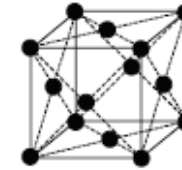
Hexagonal P



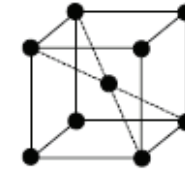
Rhombohedral P



Cubic P



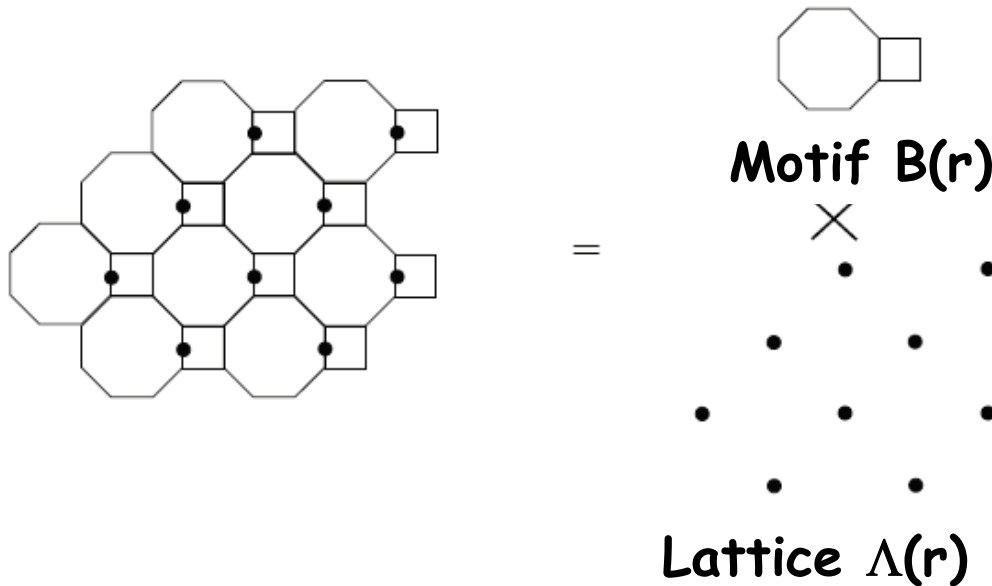
Cubic F



Cubic I

# Motif and crystal lattice:

↪ The crystal structure  $C(r)$  can be considered to arise from the convolution of a basis domain  $B(r)$  also called the motif with the Bravais lattice  $\Lambda(r)$  :  $C(r) = B(r) \times \Lambda(r)$ .



The motif  $B(r)$  = smallest possible unit of atoms which by application of all translational symmetry generates the whole crystal

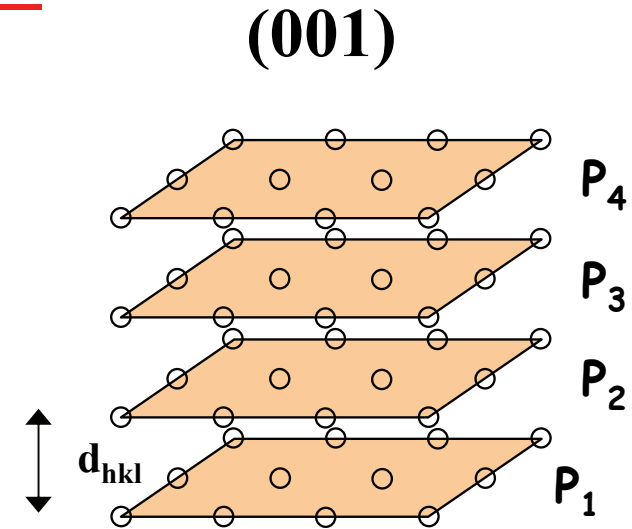
↪ The lattice  $\Lambda(r)$  can be viewed as a set of periodically spaced Dirac distributions, i.e. a Dirac  $\delta(\vec{r}-\vec{r}_0)$  distribution located on each node of the lattice:

$$\Lambda(\vec{r}) = \sum_{u=-\infty}^{+\infty} \sum_{v=-\infty}^{+\infty} \sum_{w=-\infty}^{+\infty} \delta(\vec{r} - u\vec{a} - v\vec{b} - w\vec{c})$$

with Direct lattice point coordinates :  $u\vec{a} + v\vec{b} + w\vec{c}$  with  $u$ ,  $v$  and  $w$  integers

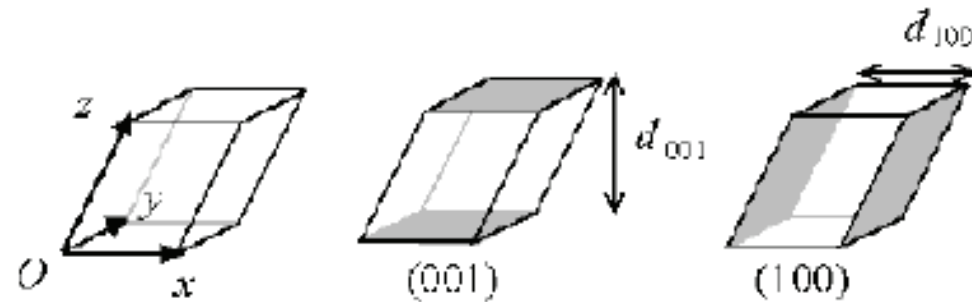
# Families of lattice planes:

↪ Lattice plane (h k l) = plane which passes through lattice nodes that do not all lie on the same straight line and labelled by three prime integer numbers h, k, l named Miller indices.



↪ Family of lattice planes = set of parallel lattice planes  $P_i$ .

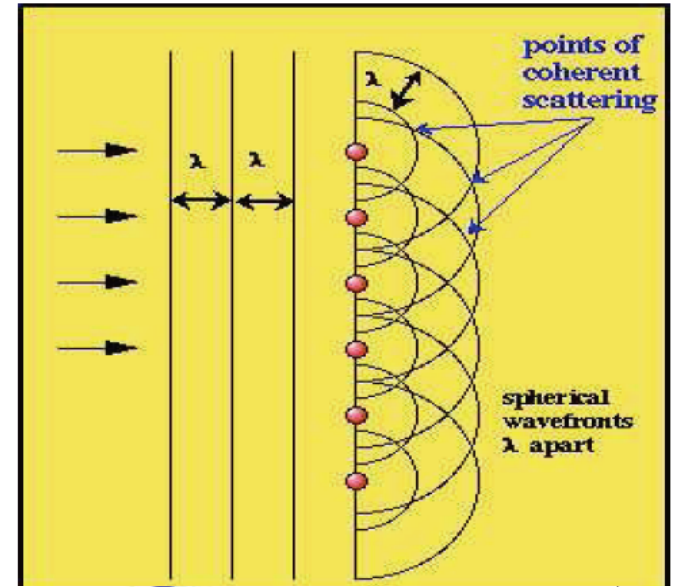
Distance between two neighbouring lattice planes = spacing  $d_{hkl}$  accessible by XRD.



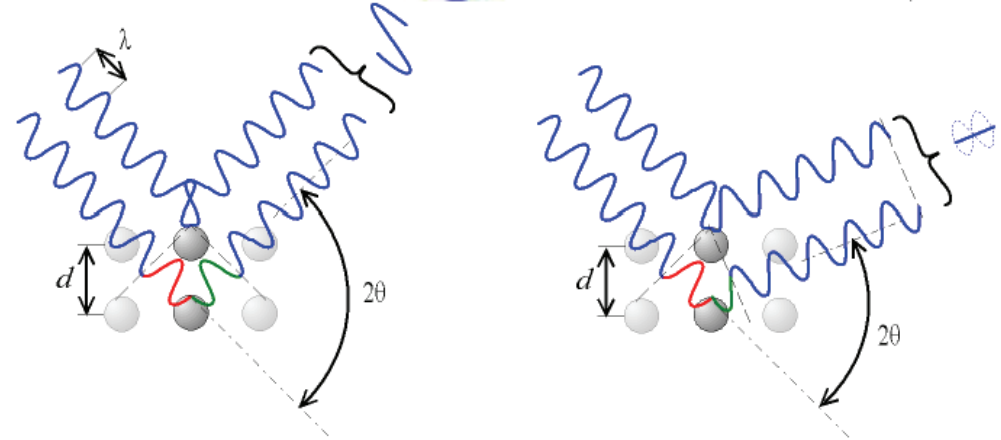
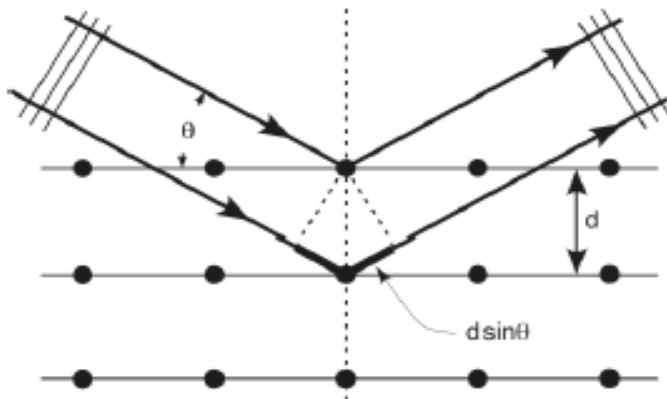
# X-ray diffraction and Bragg law

↪ When X-rays interact with a single particle, it scatters the incident beam uniformly in all directions (spherical wave).

↪ When X-rays interact with a solid material the scattered beams can add together in a few directions and reinforce each other to yield diffraction.



**Diffraction =  
Elastic diffusion +  
constructive interferences**



**Constructive interference : Path length difference  $2 d \sin \theta = \text{integral number of } \lambda$**

**Bragg law :**  
 **$2 d \sin \theta = m \lambda$  with  $m$  integer**

# Reciprocal space: Geometric definition

Introduced by Bravais and used again by Ewald (1917) : an essential concept for the study of crystal lattices and **their diffraction properties.**

- Basic vector definitions:

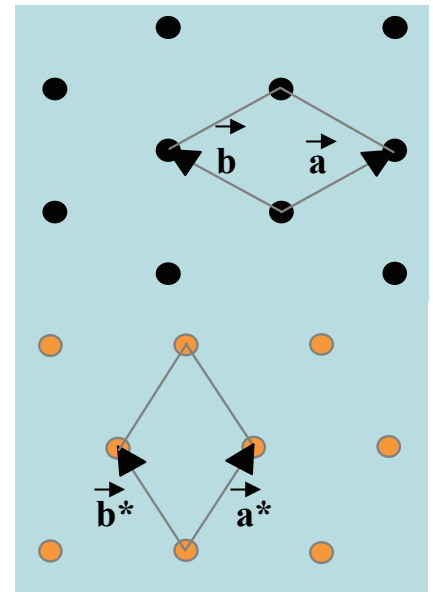
$$\vec{a}^* = \frac{\vec{b} \wedge \vec{c}}{V}, \quad \vec{b}^* = \frac{\vec{c} \wedge \vec{a}}{V}, \quad \vec{c}^* = \frac{\vec{a} \wedge \vec{b}}{V}$$

with  $V=(a,b,c)$  direct cell volume  
and  $V^*=(a^*,b^*,c^*)=1/V$  reciprocal cell volume

- Equivalent definitions (2D, 3D...)

$$\begin{array}{lll} \vec{a}^* \cdot \vec{a} = 1 & \vec{b}^* \cdot \vec{a} = 0 & \vec{c}^* \cdot \vec{a} = 0 \\ \vec{a}^* \cdot \vec{b} = 0 & \vec{b}^* \cdot \vec{b} = 1 & \vec{c}^* \cdot \vec{b} = 0 \\ \vec{a}^* \cdot \vec{c} = 0 & \vec{b}^* \cdot \vec{c} = 0 & \vec{c}^* \cdot \vec{c} = 1 \end{array}$$

Direct  
space

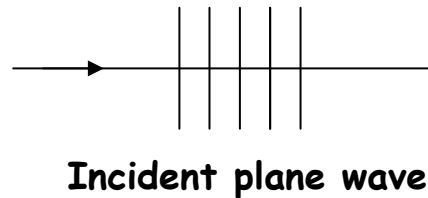


Reciprocal  
space

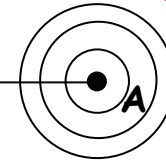
- Reciprocal lattice point coordinates:  $\vec{Q}_{hkl} = n(h\vec{a}^* + k\vec{b}^* + l\vec{c}^*)$  with  $h, k, l$  and  $n$  integers

# X-ray diffraction by crystals

## ↪ Elastic diffusion by 1 electron:



the electron (in A) acts as a secondary X-ray source



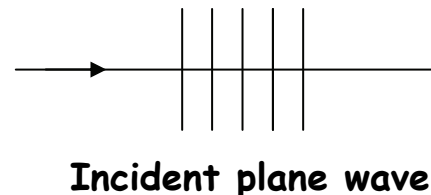
Amplitude diffused by electron  $A_e$  : proportional to  $1/m^2, q^4, \omega^4, \sin\theta$   
(Thomson formula)

## ↪ Elastic diffusion by 1 atom = N electrons

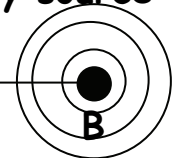
All the N spherical diffused electromagnetic waves with same phase must interfere



amplitude diffused by  
1 atom :  $A_a = f(\theta) A_e$   
with  $f(\theta)$  form factor or  
atomic diffusion coefficient



All the electrons of B atom act as a secondary X-ray source



# X-ray diffraction by crystals

## ↪ Elastic diffusion by 1 electron:



the electron (in A) acts as a secondary X-ray source

Emitted spherical wave by the electron

Amplitude diffused by electron  $A_e$  : proportional to  $1/m^2, q^4, \omega^4, \sin\theta$   
(Thomson formula)

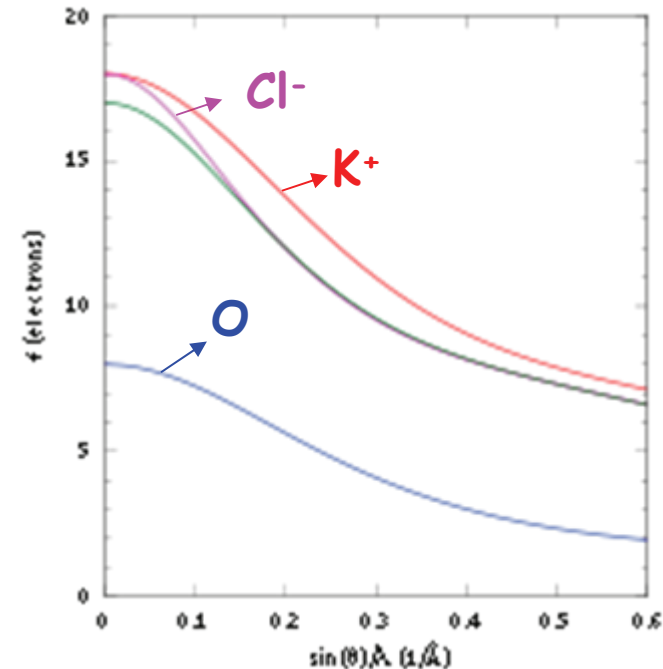
## ↪ Elastic diffusion by 1 atom = N electrons

All the N spherical diffused electromagnetic waves with same phase must interfere



amplitude diffused by  
1 atom :  $A_a = f(\theta) A_e$   
with  $f(\theta)$  form factor or  
atomic diffusion coefficient

Form factor  $f(\sin\theta/\lambda)$  tabulated in the International Crystallography Tables



$f(\theta) = Z$  for  $\theta = 0^\circ$  +  $f(\theta)$   
fall down rapidly when  $\theta \nearrow$

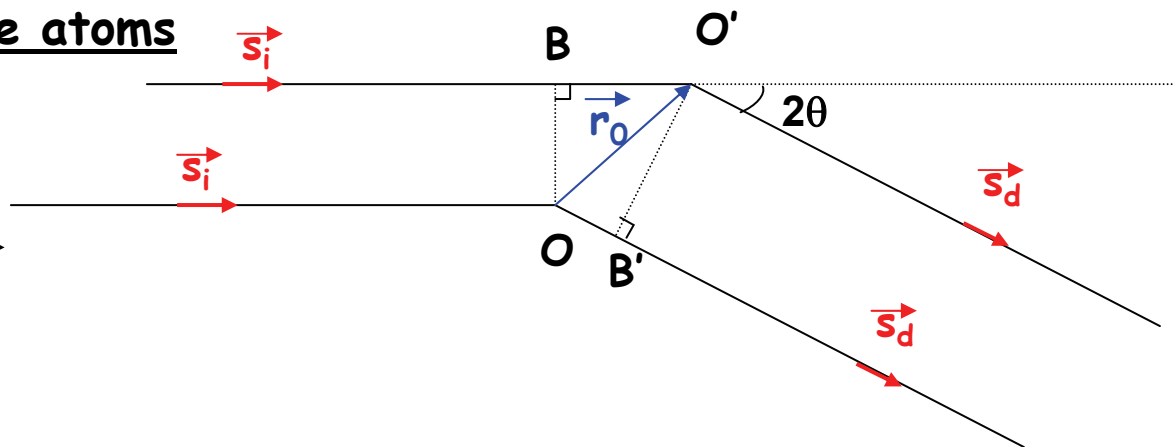
# X-ray diffraction by crystals

## Elastic diffusion by 2 same atoms

$\vec{s}_i$  and  $\vec{s}_d$ : unitary incident and diffused vectors

$$OB' = \vec{r}_0 \cdot \vec{s}_d \text{ et } O'B = \vec{r}_0 \cdot \vec{s}_i$$

$$\delta = \vec{r}_0 \cdot (\vec{s}_d - \vec{s}_i)$$



The path difference  $\delta$  between the two scattered waves:  $\delta = OB' - O'B$  gives the phase difference  $\Delta\varphi$  between these 2 waves:  $\Delta\varphi = 2\pi \delta / \lambda$

$$\Delta\varphi = \frac{2\pi}{\lambda} \vec{r}_0 \cdot (\vec{s}_d - \vec{s}_i) = 2\pi \vec{r}_0 \cdot \vec{H} \quad \text{avec } \vec{H} = (\vec{s}_d - \vec{s}_i) / \lambda : \text{diffusion vector}$$

$$\text{Resulting wave amplitude : } A = A_a (1 + e^{2j\pi(\vec{r}_0 \cdot \vec{H})}) = f A_e (1 + e^{2j\pi(\vec{r}_0 \cdot \vec{H})})$$

Constructive interference if  $\Delta\varphi = 2\pi n$  with n integer  $\rightarrow \vec{r}_0 \cdot \vec{H} = n$

et  $\vec{r}_0 = u\vec{a} + v\vec{b} + w\vec{c} \rightarrow 3$  relations that must be verified simultaneously

$$\begin{aligned} \vec{a} \cdot \vec{H} &= h \\ \vec{b} \cdot \vec{H} &= k \\ \vec{c} \cdot \vec{H} &= l \end{aligned}$$

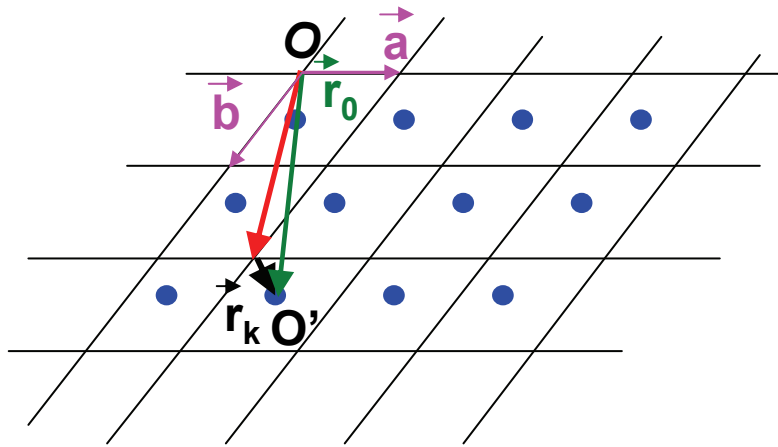
Laue conditions:

Constructive interference i.e. diffraction if diffusion vector  $\vec{H}$  is a reciprocal space vector



Elastic diffusion by a monoatomic crystal

Diffused amplitude by O' atom with the phase origin in O:  $f A_e e^{2j\pi(\vec{r}_0 \cdot \vec{H})}$



Motif = • = 1 atom with (x, y, z) coordinates :  $\vec{r}_k = x_k \vec{a} + y_k \vec{b} + z_k \vec{c}$

$$\vec{r}_0 = \underbrace{\vec{r}_k}_{\text{motif}} + u \vec{a} + v \vec{b} + w \vec{c}$$

lattice

Crystal = n atoms : n diffused waves with different phase;  
Diffused amplitude by n atoms :

$$\mathbf{A} = \sum_{k=1}^n f A_e e^{2j\pi(\vec{r}_0 \cdot \vec{H})} = \sum_{k,u,v,w} f A_e e^{2j\pi(\vec{r}_k \cdot \vec{H})} e^{2j\pi[u(\vec{H} \cdot \vec{a}) + v(\vec{H} \cdot \vec{b}) + w(\vec{H} \cdot \vec{c})]}$$

Diffraction if  $\mathbf{A} \neq 0$  so  $\vec{H} \cdot \vec{a}, \vec{H} \cdot \vec{b}$  and  $\vec{H} \cdot \vec{c} = \text{integers}$ .

$$\mathbf{A} = A_e \sum_k f_k e^{2j\pi(\vec{r}_k \cdot \vec{H})} \times u \times v \times w = N A_e \sum_k f_k e^{2j\pi(\vec{r}_k \cdot \vec{H})}$$

Structure factor  $F = A/(N A_e)$

$$F_{hkl} = \sum_k f_k e^{2j\pi(\vec{r}_k \cdot \vec{H})} = \sum_k f_k e^{2j\pi(hx_k + ky_k + lz_k)}$$

Formula that can be generalized also for polyatomic crystals

## ↳ Temperature factor = Debye-Waller factor

When  $\theta \nearrow$ , atoms vibration amplitude  $\nearrow$ : 
$$F_{hkl} = \sum_{\mathbf{k}} f_{\mathbf{k}} e^{2j\pi(hx_{\mathbf{k}} + ky_{\mathbf{k}} + lz_{\mathbf{k}})} \times e^{\left(\frac{-B \sin^2 \theta}{\lambda}\right)}$$

with  $(x_{\mathbf{k}}, y_{\mathbf{k}}, z_{\mathbf{k}})$  = atomic position in the unit cell and  $B = 8\pi^2 \langle u^2 \rangle$

$\langle u^2 \rangle$  = standard deviation of the amplitude of atom vibration  
↓

The influence of  $B$  is more important when  $\theta \nearrow$ , i.e. when diffracted beam is far from the incident beam direction.

↳ **Intensity** : the diffracted intensity in the direction  $\vec{s}_d = \vec{s}_i + \lambda \vec{H}$

is proportional to  $FF^*$ :  $I_{hkl} \propto N^2 A_e F_{hkl} F_{hkl}^*$

with 
$$F_{hkl}^* = \sum_{\mathbf{k}} f_{\mathbf{k}} e^{-2j\pi(hx_{\mathbf{k}} + ky_{\mathbf{k}} + lz_{\mathbf{k}})} \times e^{\left(\frac{-B \sin^2 \theta}{\lambda}\right)}$$

Friedel law:  $F_{hkl}^* = F(\bar{h} \bar{k} \bar{l})$  and  $F(\bar{h} \bar{k} \bar{l}) = F_{hkl}^* \rightarrow I(\bar{h} \bar{k} \bar{l}) = I(hkl)$   
if  $f_{\mathbf{k}}$  is real : we cannot know if a structure is centrosymmetric or not !

↳ **Systematic extinctions (due to the lattice) :  $I(hkl) = 0$**

\* **For a body centered lattice (I)** : additional translation  $\vec{t} = \left( \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c} \right)$

To each atom in  $x_k, y_k, z_k$  corresponds systematically an atom in  $x_k+1/2, y_k+1/2, z_k+1/2$

$$F_{hkl} = \left( \sum_{\text{half atoms}} f_k e^{2j\pi(hx_k + ky_k + lz_k)} + \sum_{\text{half atoms deduced by } \vec{t}} f_k e^{2j\pi\left(h\left(x_k + \frac{1}{2}\right) + k\left(y_k + \frac{1}{2}\right) + l\left(z_k + \frac{1}{2}\right)\right)} \right) \times e^{\left(\frac{-B \sin^2 \theta}{\lambda}\right)}$$

$$F_{hkl} = \sum_{\text{half atoms}} f_k e^{2j\pi(hx_k + ky_k + lz_k)} \left[ 1 + e^{\frac{h+k+l}{2}} \right] \times e^{\left(\frac{-B \sin^2 \theta}{\lambda}\right)}$$

motif

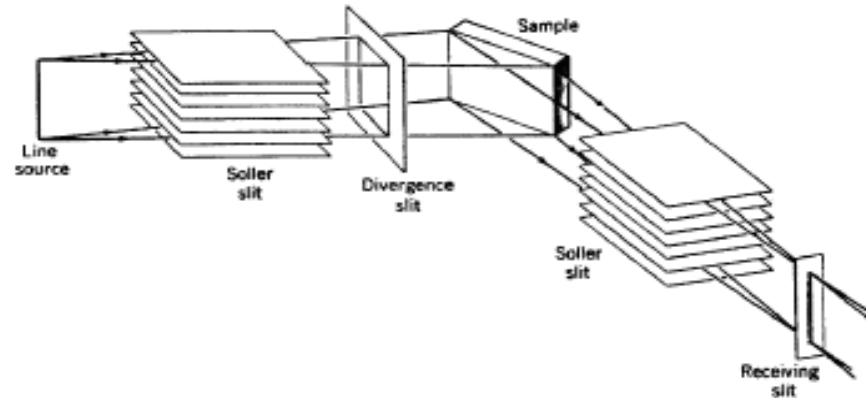
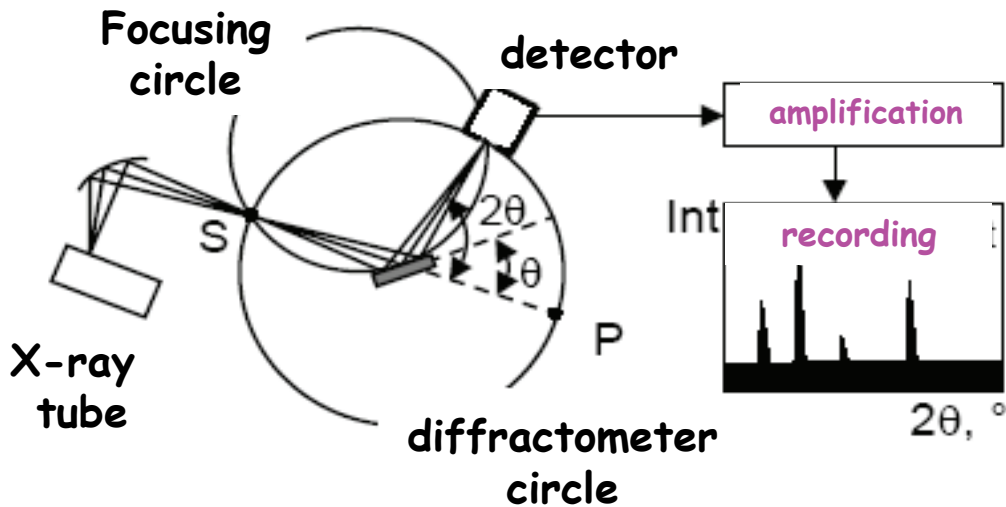
lattice

If  $h+k+l$  odd,  $F(hkl) = 0$  :  
systematic extinctions due to the I lattice if  $h+k+l=2n+1$ .

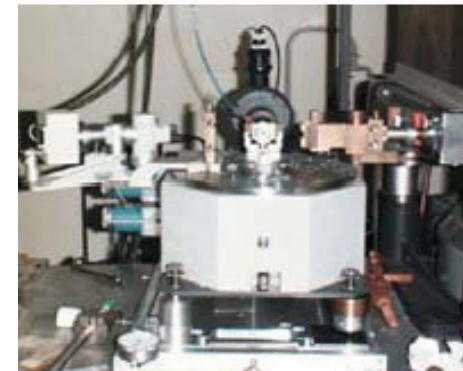
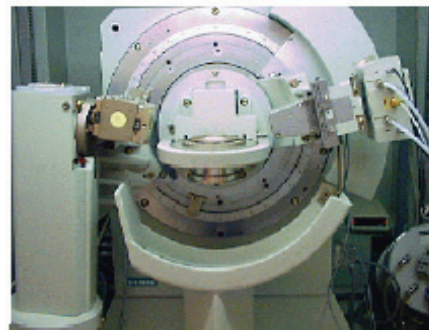
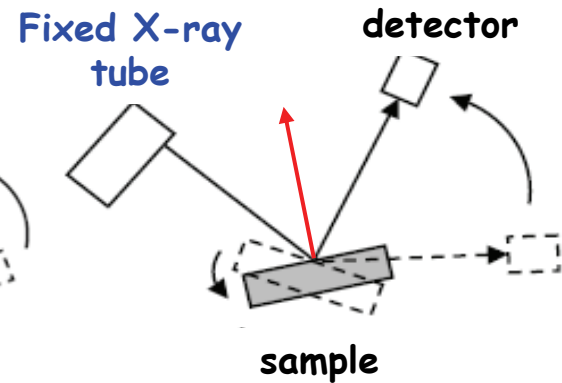
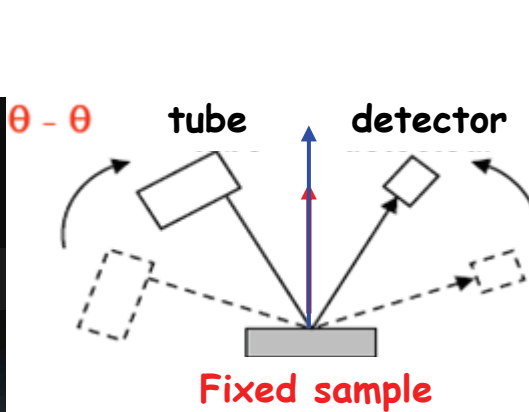
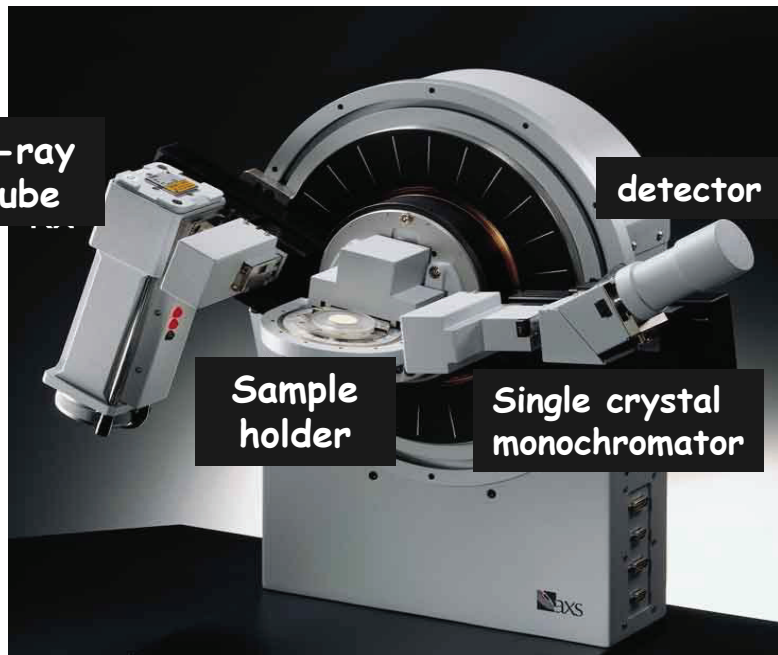
\* **For a face centered lattice (F)** : 3 additional translations  $\vec{t} = \left( \frac{1}{2}\vec{a} + \frac{1}{2}\vec{b} \right)$   
 $\vec{t} = \left( \frac{1}{2}\vec{a} + \frac{1}{2}\vec{c} \right)$  and  $\vec{t} = \left( \frac{1}{2}\vec{b} + \frac{1}{2}\vec{c} \right)$  ; extinctions if  $h, k$  and  $l$  none of same parity.

\* **For primitive lattice (P)** : no extinction.

# Powder diffractometer (2-circles)



## Bragg Brentano geometry

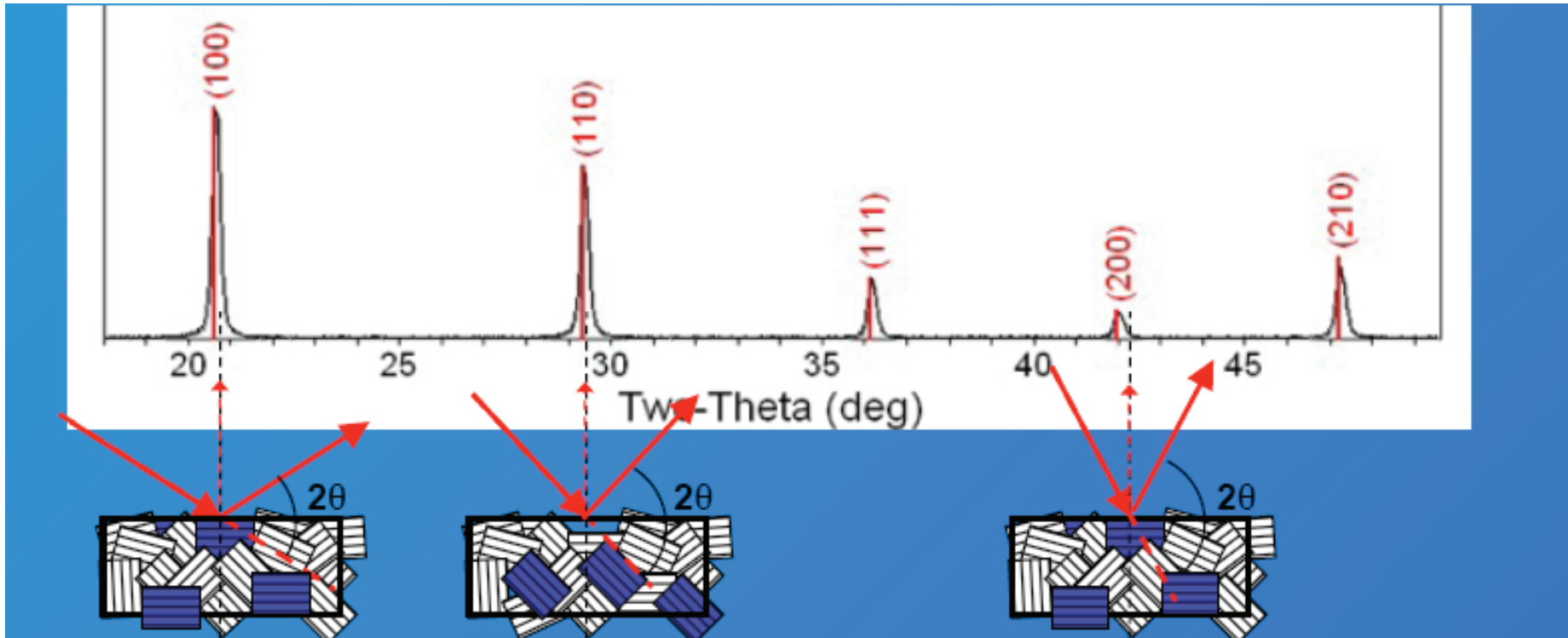


Polycrystalline sample = n single crystals

If n single crystals with random orientation = Powder



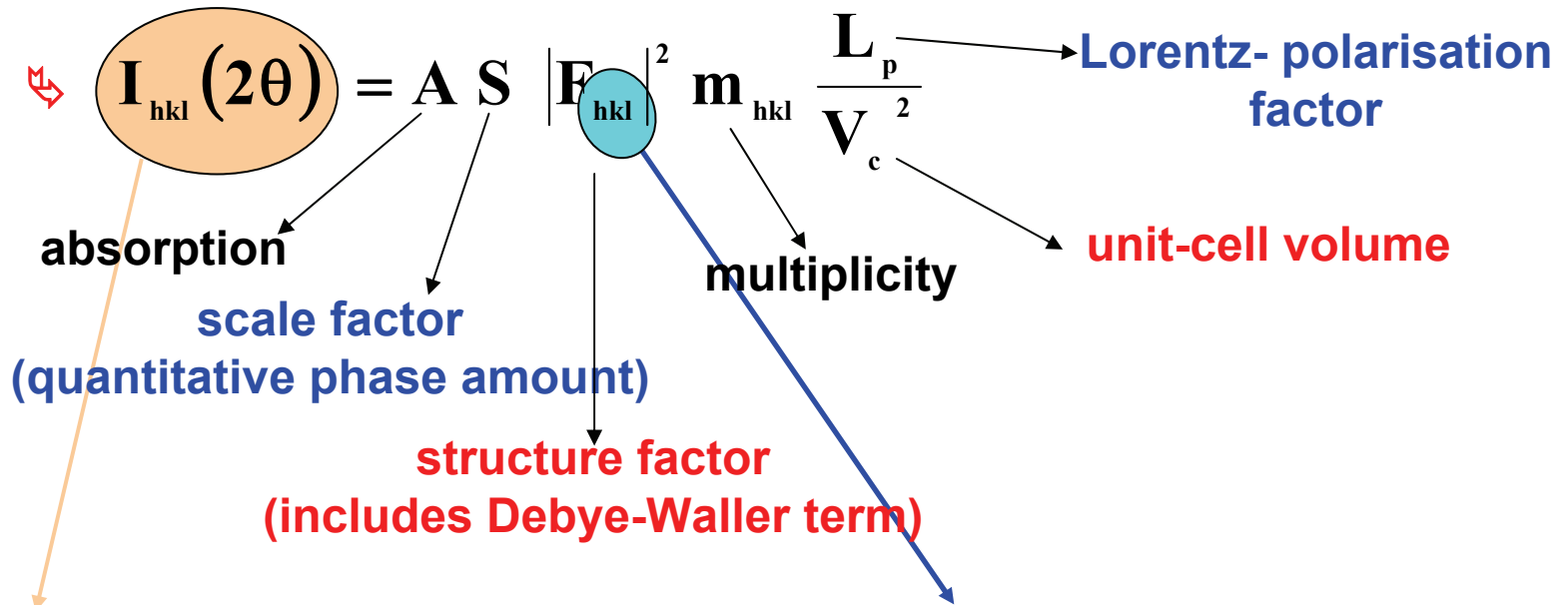
All possible diffraction peaks (Bragg law) are observed



For every set of planes parallel to the sample surface, there will be some crystallites that are properly oriented to diffract (i.e. plane  $\perp$  bisects the incident and diffracted beams).

Basic assumptions of powder diffraction = for every set of planes there is an equal number of crystallites that will diffract.

Random powder :  $I_{RX \text{ calc.}}(2\theta) = \sum_{hkl, \text{phases}} I_{hkl, \text{phases}}(2\theta) S_{hkl}(2\theta) + \text{bkg}(2\theta)$



which atoms are ("motif");  
Number of scattering electron

$d_{hkl}$  : where the atoms are: due to "lattice";  
position in reciprocal space

**Random powder** :  $I_{RX \text{ calc.}}(2\theta) = \sum_{hkl, \text{phases}} I_{hkl, \text{phases}}(2\theta) S_{hkl}(2\theta) + \text{bkg}(2\theta)$

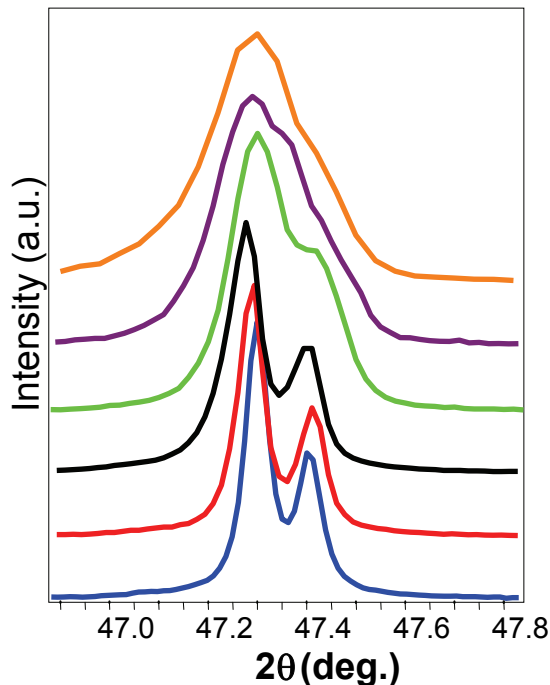
$S_{hkl}(2\theta) = S_{hkl}^I(2\theta) * S_{hkl}^S(2\theta)$

Peak profile function

Sample aberrations = crystallite sizes (isotropic or anisotropic)

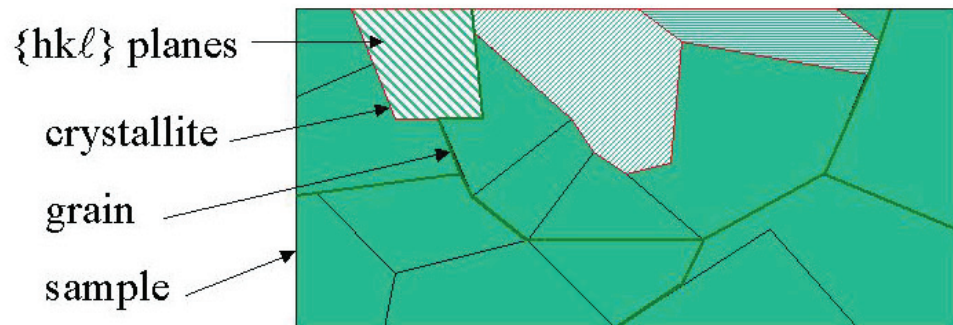
instrumental broadening → powder standard calibration

+ rms microstrains  $\epsilon^* = \langle \epsilon \rangle^2$   
(Non-uniform Lattice Distortions, Faulting, Dislocations, Antiphase Domain Boundaries ...)

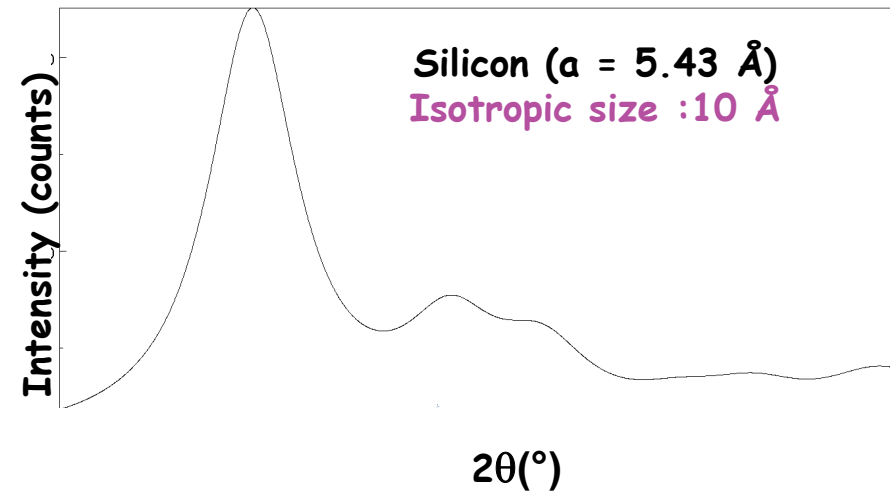
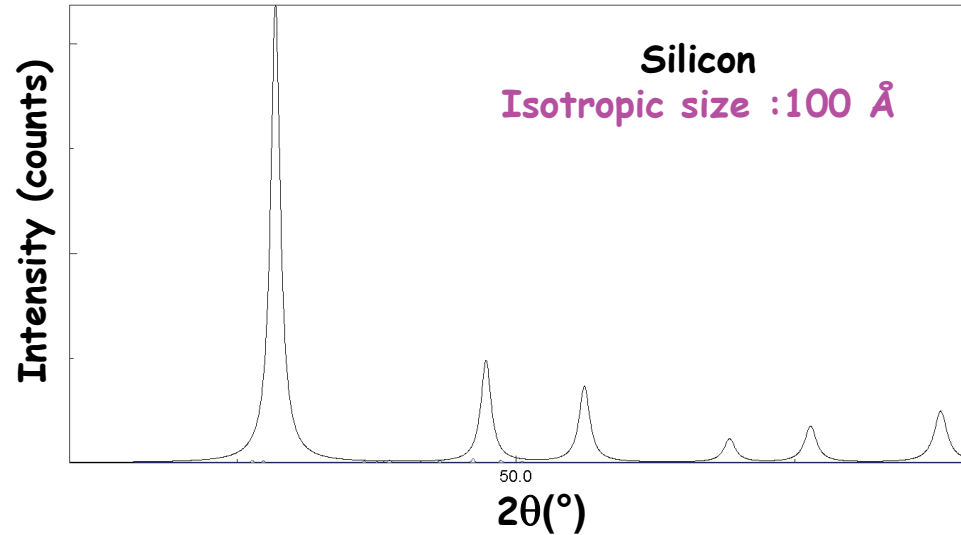
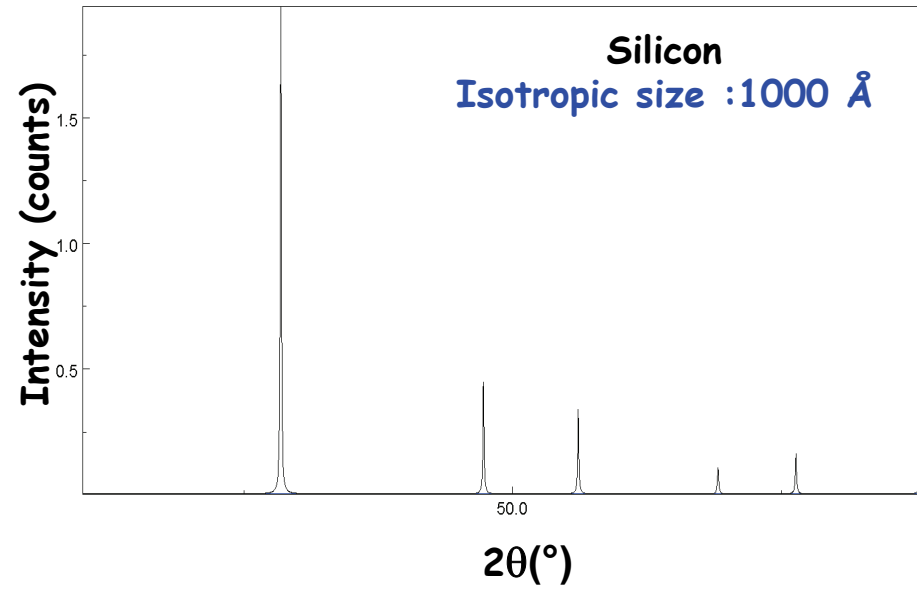
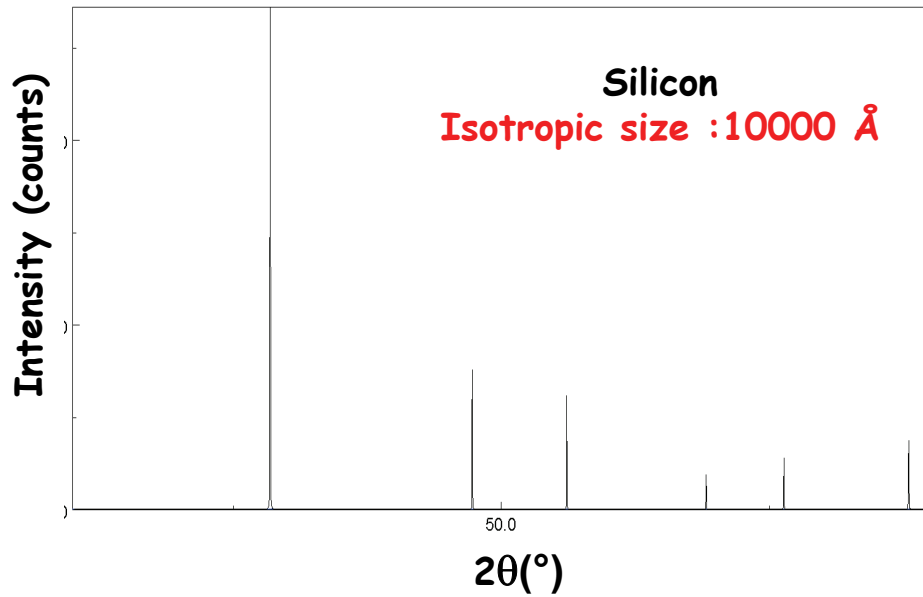


Patterns collected from the same sample with different instruments and configurations

**Take care:**  
crystallite size is different than grain size !!!



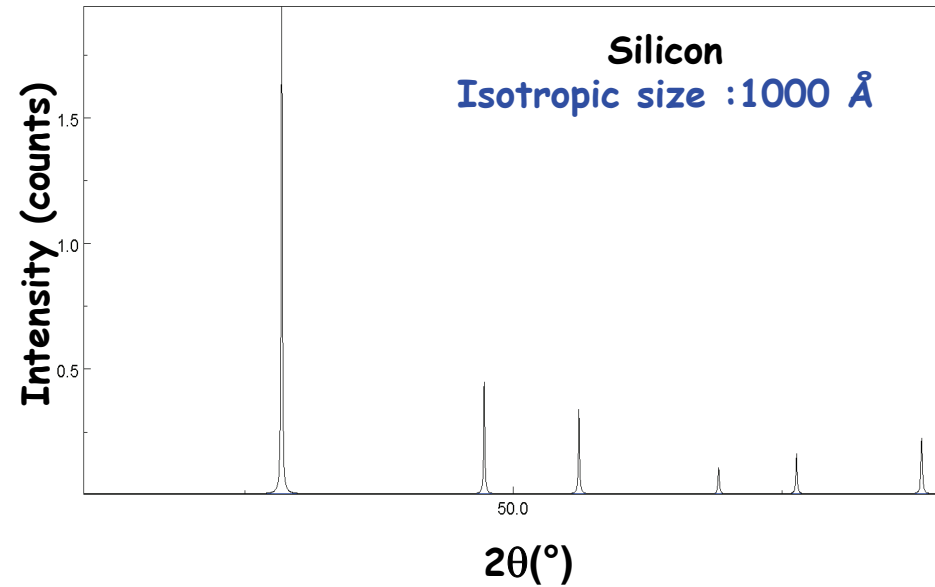
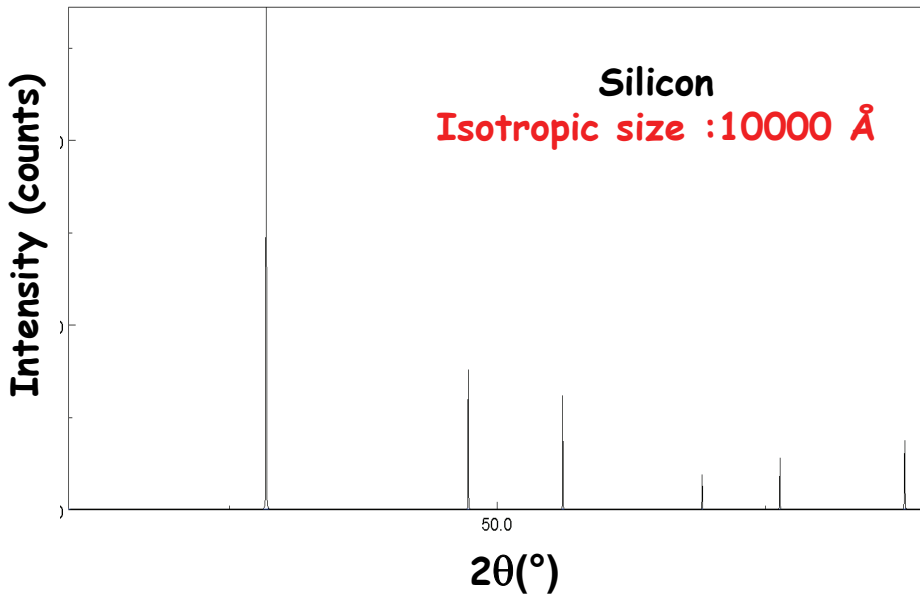
A grain may be made up of several different crystallites



Peak width (B) is inversely proportional to crystallite size (L) →  
Debye Scherrer formula : estimation of crystallite size

$$B(2\theta) = \frac{K\lambda}{L \cos \theta}$$





The constant of proportionality,  $K$  (Scherrer constant) depends on how the width is determined, the shape of the crystal, and the size distribution.

The most common values for  $K$  are:

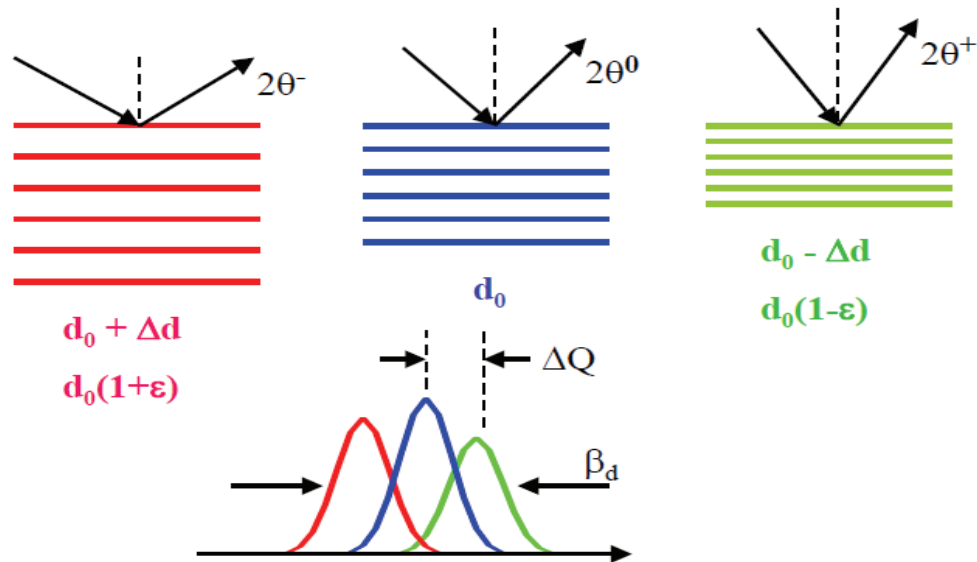
- 0.94 for FWHM of spherical crystals
- 0.89 for integral breadth of spherical crystals

Peak width ( $B$ ) is inversely proportional to crystallite size ( $L$ ) →  
Debye Scherrer formula : estimation of crystallite size

$$B(2\theta) = \frac{K\lambda}{L \cos \theta}$$

## Peak broadening by microstrain (varying lattice parameter = non uniform strain)

Microstrain :  $\langle \varepsilon \rangle = \frac{\Delta d}{d}$  ; in a same grain (several crystallites) :



Pic width : effect of microstrain = peaks juxtaposition :

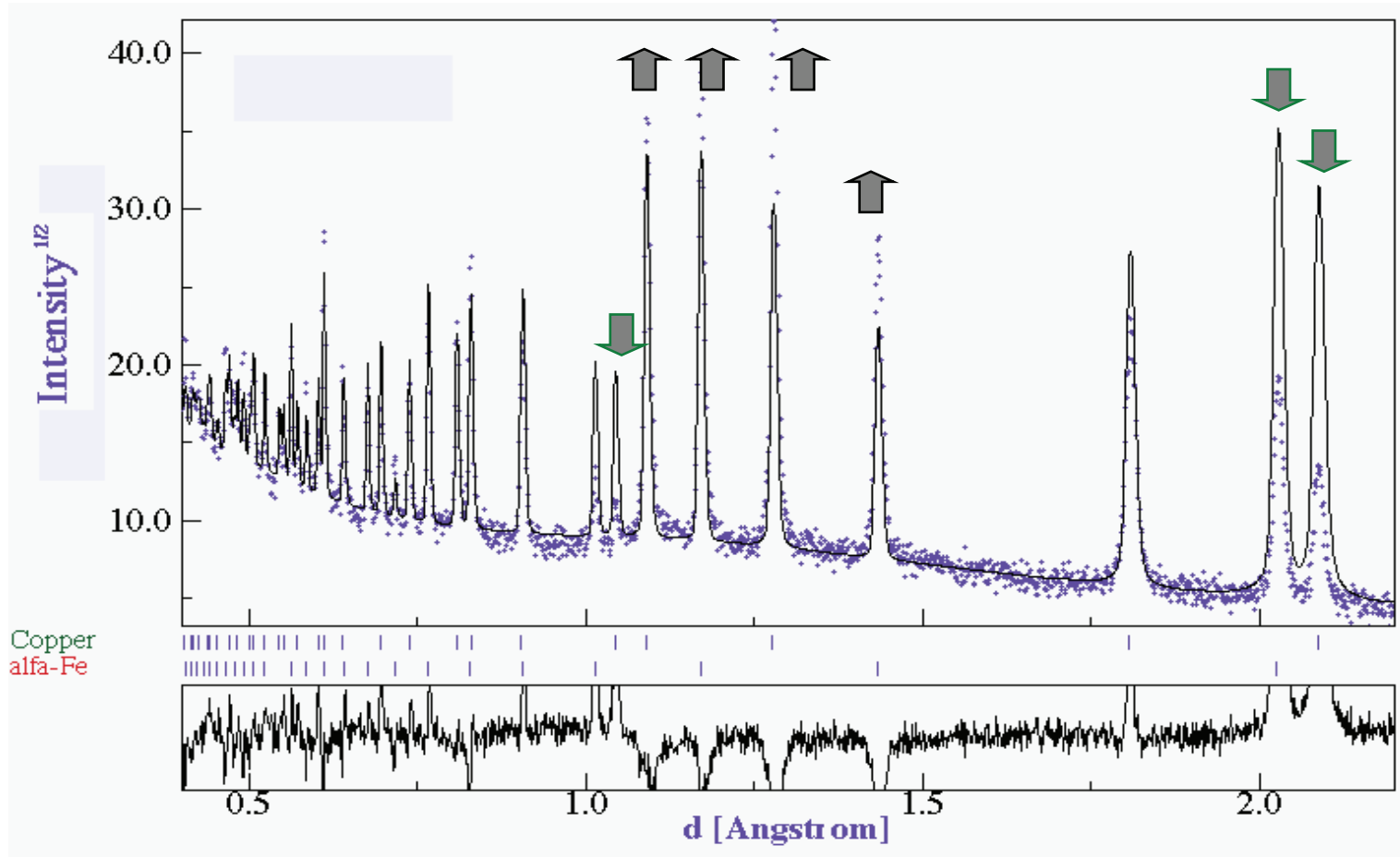


Increased microstrain results in broadening of diffraction peaks

If n single crystals with specific orientation = textured sample



Not all possible diffraction peaks are observed or all possible diffraction peaks are observed but with different intensities compared to a powder



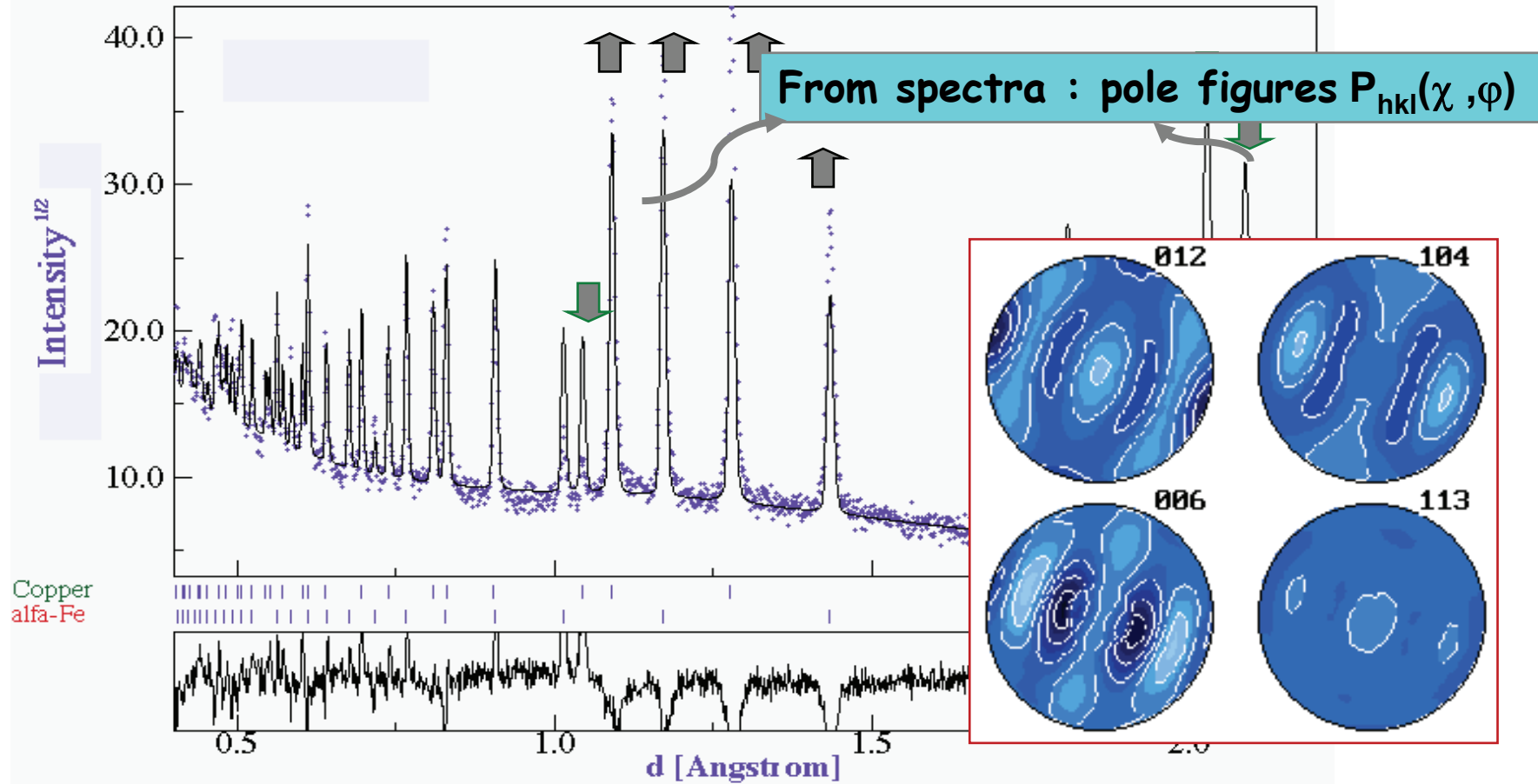
Correction of intensities for textured sample :

$$I_{hkl}(2\theta, \chi, \varphi) = I_{hkl}(2\theta) P_{hkl}(\chi, \varphi)$$

If n single crystals with specific orientation = textured sample



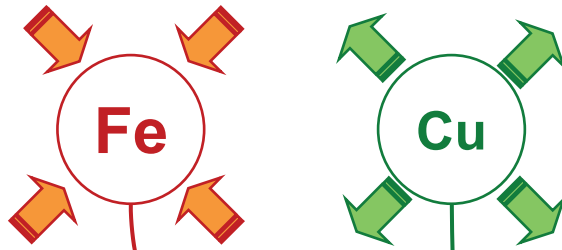
Not all possible diffraction peaks are observed or all possible diffraction peaks are observed but with different intensities compared to a powder



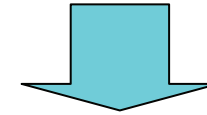
Correction of intensities for textured sample :

$$I_{hkl}(2\theta, \chi, \varphi) = I_{hkl}(2\theta) P_{hkl}(\chi, \varphi)$$

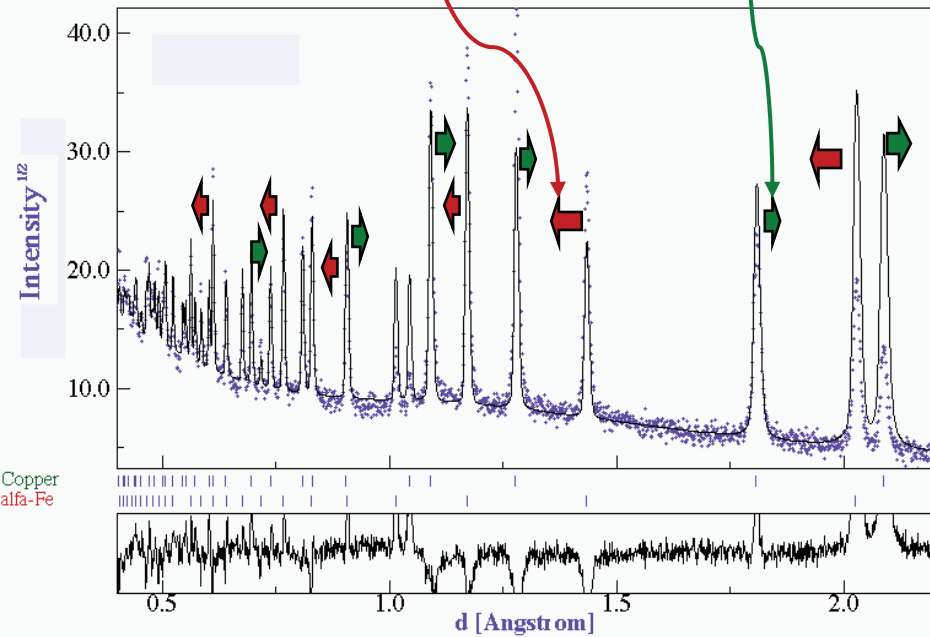
## Residual stresses



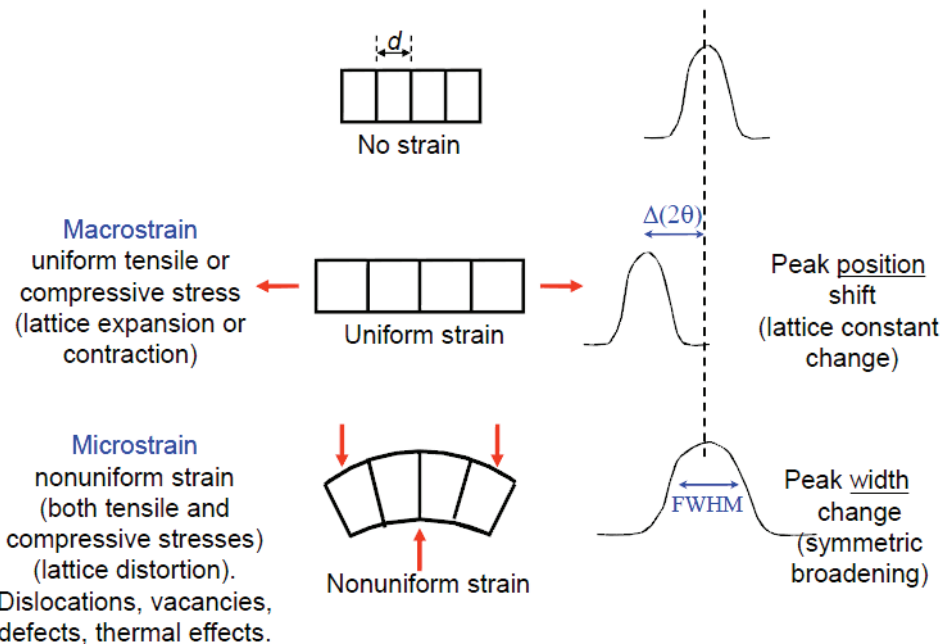
By changing the orientation of the sample towards the incident beam : peak position moves from a reference position but intensity is the same



peak shifts due to lattice expansion or contraction → tensile or compressive stress



Copper  
alpha-Fe



## Summary

# Summary

You can use XRD on powder polycrystalline samples to determine:

- **Phase Composition of the sample**
  - *Quantitative Phase Analysis: determine the relative amounts of phases in a mixture by referencing the relative peak intensities*
- **Unit cell lattice parameters and Bravais lattice symmetry**
  - *Index peak positions*
  - *Lattice parameters can vary as a function of, and therefore give you information about, alloying, doping, solid solutions ...*
- **Residual Strain (macrostrain)**
- **Texture/Orientation**
- **Crystal Structure**
  - *By Rietveld refinement of the entire diffraction pattern (not presented here)*
- **Crystallite Size and microstrain**
  - *Indicated by peak broadening*
  - Other defects (stacking faults, etc.) can be measured by analysis of peak shapes and peak width.*